Game Theory and Gricean Pragmatics

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The course

- concentrates on **Gricean Pragmatics**,
- is concerned with the foundation of pragmatics on **Lewis (1969) theory of Conventions**,
- uses **classical game theory**.

Course Overview

- Lesson 1: Introduction
  - From Grice to Lewis
  - Relevance Scale Approaches
- Lesson 2: Signalling Games
  - Lewis‘ Signalling Conventions
  - Parikh‘s Radical Underspecification Model
- Lesson 3: The Optimal Answer Approach I
- Lesson 4: The Optimal Answer Approach II
  - Decision Contexts with Multiple Objectives
  - Comparison with Relevance Scale Approaches

Game Theory and Gricean Pragmatics Lesson I

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From Grice to Lewis

Lesson 1 – April, 4th

Overview of Lesson I

- Gricean Pragmatics
  - General assumptions about conversation
  - Conversational implicatures
- Game and Decision Theory
- Relevance Scale Approaches
  - An Argumentative View: A. Merin
  - A Non-Argumentative View: R. v. Rooij

Gricean Pragmatics

General assumptions about conversation

Gricean Pragmatics

Grice distinguishes between:

- What is **said**.
- What is **implicated**.

“Some of the boys came to the party.”

- **said:** At least two of the boys came to the party.
- **implicated:** Not all of the boys came to the party.

Both part of what is **communicated**.

Assumptions about Conversation

- Conversation is a **cooperative effort**.
- Each participant recognises in the talk exchange a **common purpose**.
- A stands in front of his obviously immobilised car.

  A: I am out of petrol.

  B: There is a garage around the corner.
- **Joint purpose of B's response**: Solve A's problem of finding petrol for his car.

The Cooperative Principle

Conversation is **governed by a set of principles** which spell out how rational agents behave in order to make language use efficient.

The most important is the so-called **cooperative principle**:

“Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged.”

The Conversational Maxims

Maxim of Quality:

1. Do not say what you believe to be false.
2. Do not say that for which you lack adequate evidence.

Maxim of Quantity:

1. Make your contribution to the conversation as informative as is required for he current talk exchange.
2. Do not make your contribution to the conversation more informative than necessary.

Maxim of Relevance:

Make your contributions relevant.

Maxim of Manner:

Be perspicuous, and specifically:

1. Avoid obscurity.
2. Avoid ambiguity.
3. Be brief (avoid unnecessary wordiness).
4. Be orderly.

The Conversational Maxims
(short, without Manner)

Maxim of Quality: Be truthful.

Maxim of Quantity:

1. Say as much as you can.
2. Say no more than you must.

Maxim of Relevance: Be relevant.

The Conversational Maxims

Be truthful (Quality) and say as much as you can (Quantity) as long as it is relevant (Relevance).

Conversational implicatures

An example: Scalar Implicatures

"Some of the boys came to the party."

- **said:** At least two of the boys came to the party.
- **implicated:** Not all of the boys came to the party.

Both part of what is communicated.

An Explanation based on Maxims

Let $A(x) \equiv$ “x of the boys came to the party”

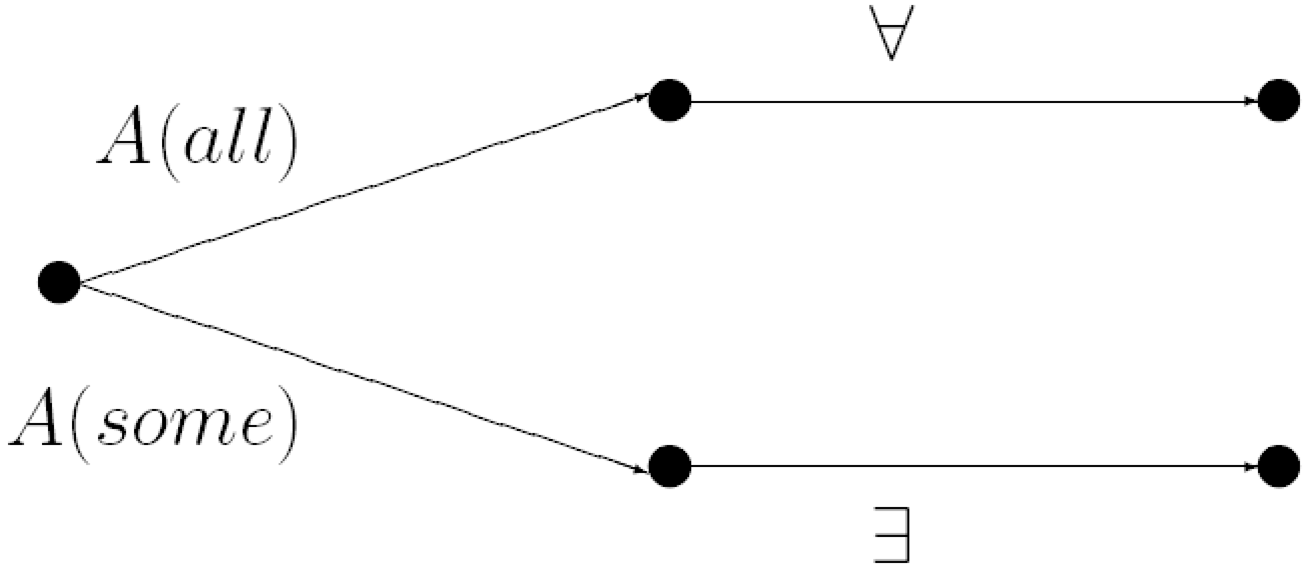
1. The speaker had the choice between the forms A(all) and A(some).
2. A(all) is more informative than A(some) and the additional information is also relevant.
3. Hence, if all of the boys came, then A(all) is preferred over A(some) (Quantity) + (Relevance).

1. The speaker said A(some).
2. Hence it cannot be the case that all came.
3. Therefore some but not all came to the party.

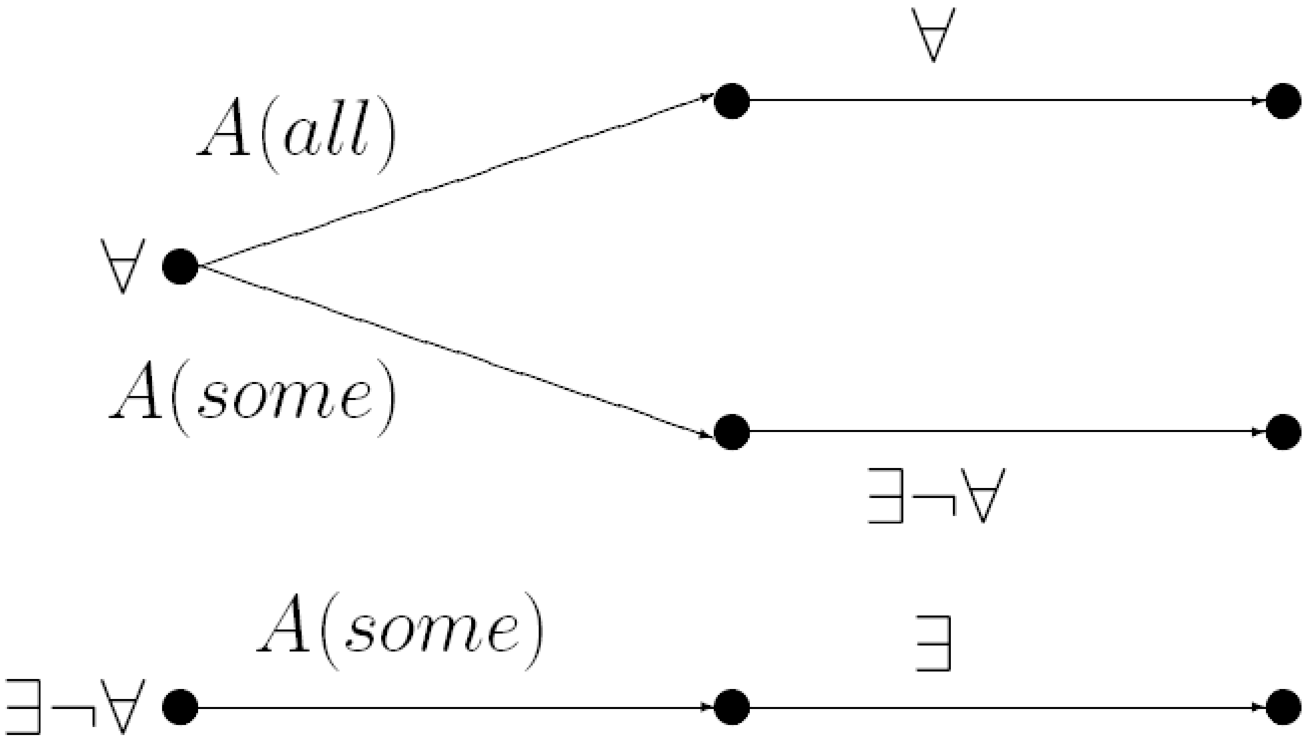
A Graphical Interpretation I

- The speaker has a choice between A(all) and A(some).
- If he chooses A(all), the hearer has to interpret ‘all’ by the universal quantifier.
- If he chooses A(some), the hearer has to interpret ‘some’ by the existential quantifier.

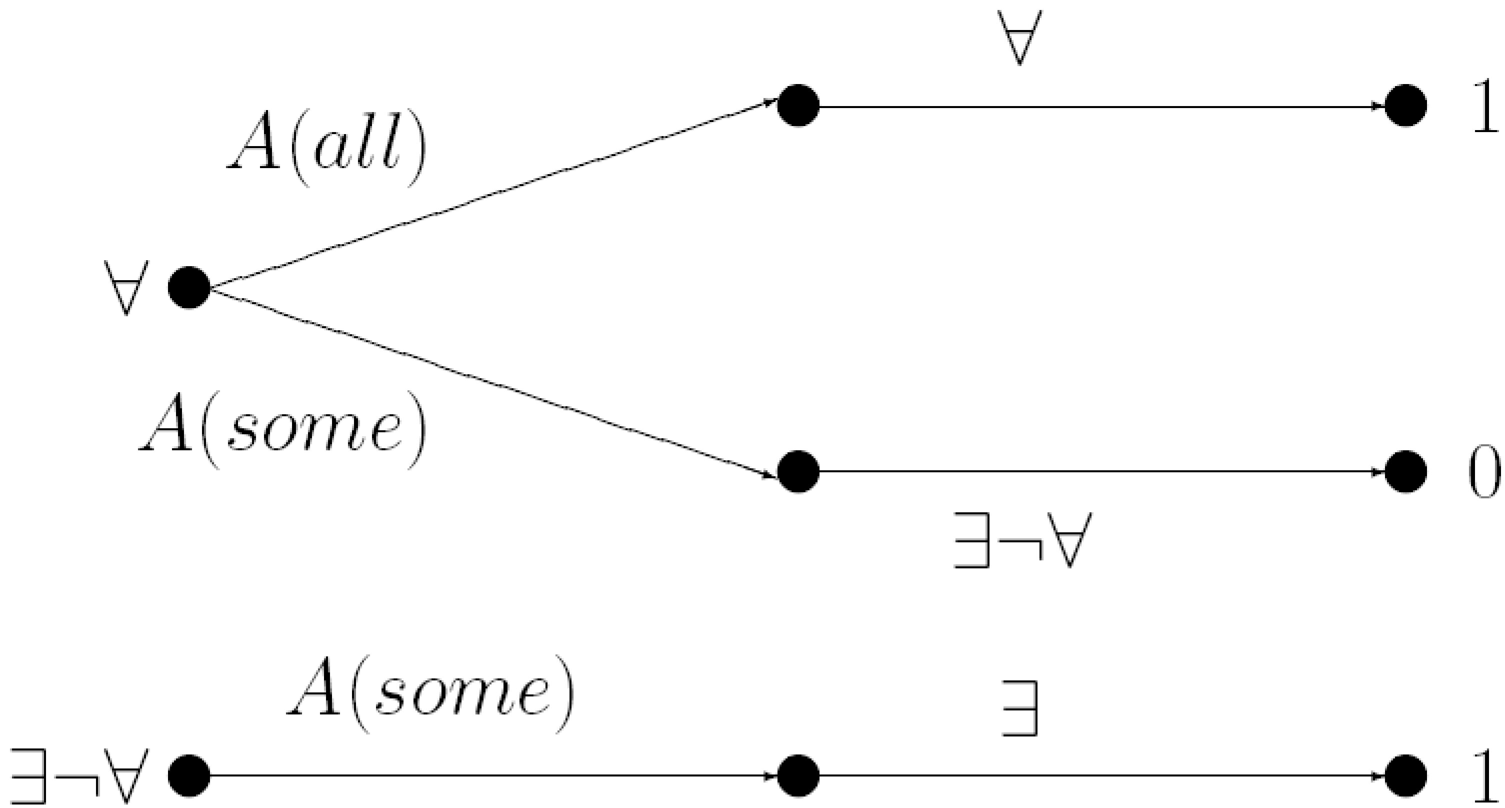
The situation were all of the boys came to the party:



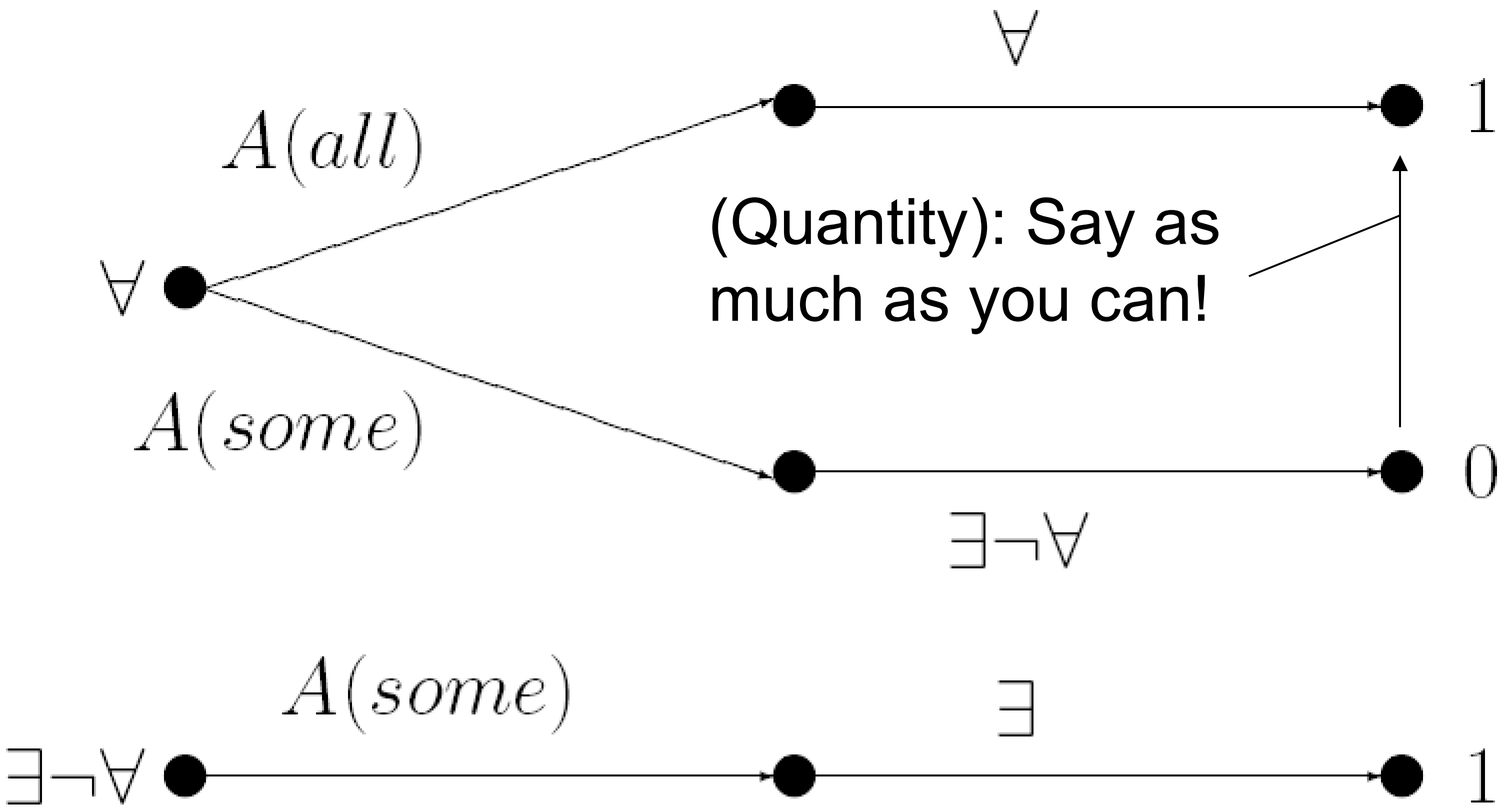
Taking into account the alternative situation where some but not all came:



Adding speaker's preferences:



Adding speaker's preferences:

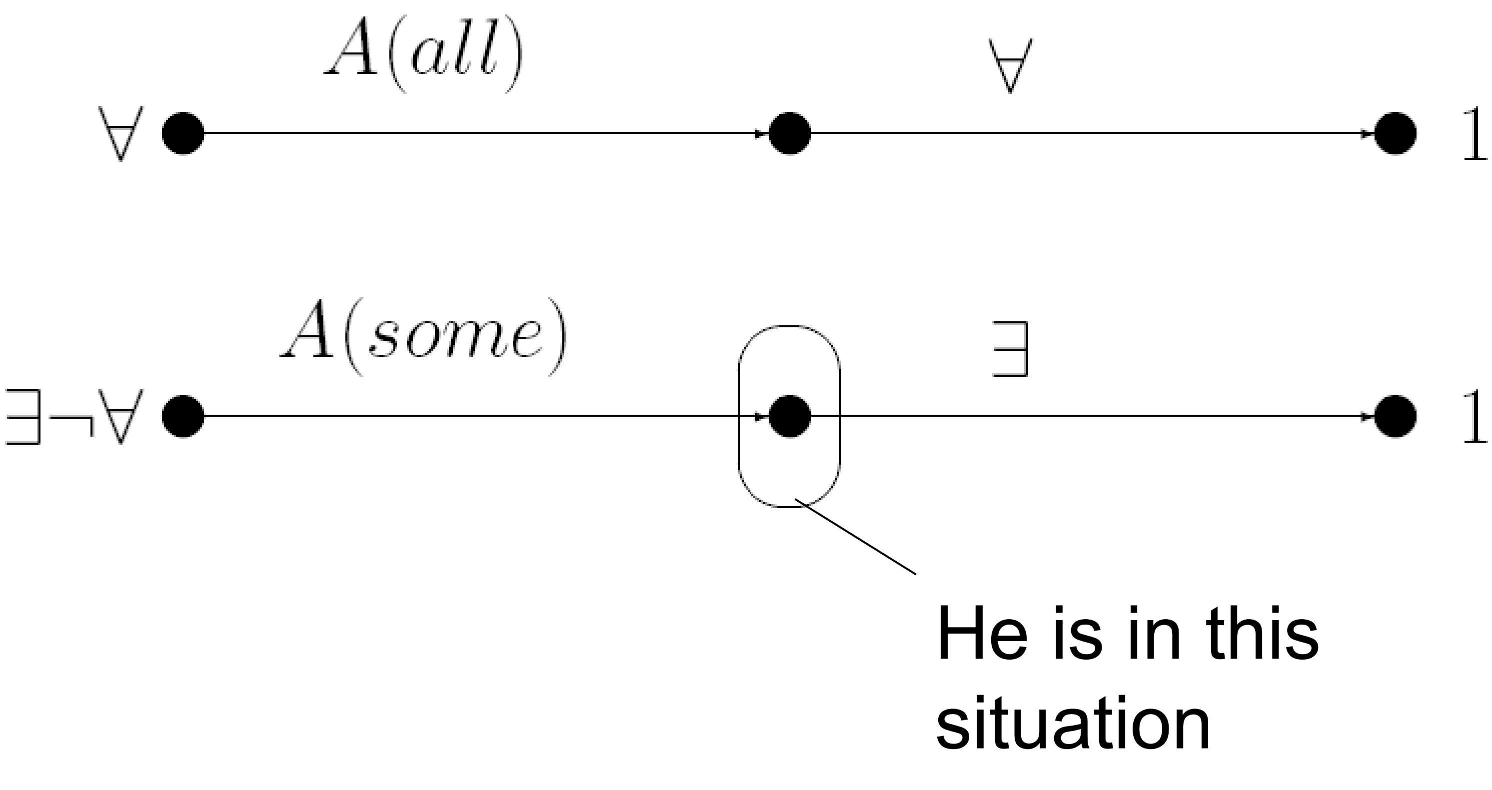


Hence, the speaker will choose:

$$\forall \; \bullet \xrightarrow{\quad A(all) \quad} \bullet \xrightarrow{\quad \forall \quad} \bullet \; 1$$

$$\exists\neg\forall \; \bullet \xrightarrow{\quad A(some) \quad} \bullet \xrightarrow{\quad \exists \quad} \bullet \; 1$$

Hence, the hearer can infer after receiving A(some) that:



Why a New Framework?

- Basic concepts of Gricean pragmatics are undefined, most notably the concept of relevance.
- On a purely intuitive level, it is often not possible to decide whether an inference of an implicatures is correct or not.

An Example

A stands in front of his obviously immobilised car.

A: I am out of petrol.

B: There is a garage around the corner. (G)

+> The garage is open (H)

A “standard” explanation

Set H*:= The negation of H

- B said that G but not that H*.
- H* is relevant and $G \wedge H^* \Rightarrow G$.
- Hence if $G \wedge H^*$, then B should have said $G \wedge H^*$ (Quantity).
- Hence H* cannot be true, and therefore H.

A Second Explanation

1. B said that G but not that H.
2. H is relevant and $G \wedge H \Rightarrow G$.
3. Hence if $G \wedge H$, then B should have said $G \wedge H$ (Quantity).
4. Hence H cannot be true, and therefore H*.

Problem: We can exchange H and H* and still get a *valid* inference.

Without clarification of its basic concepts, the theory of conversational implicatures lacks true predictive power.

Game and Decision Theory

Game and Decision Theoretic Approaches to Gricean Pragmatics

Distinguish between Approaches based on:

- Classical Game Theory
  - Underspecification based Approach (P. Parikh).
  - Optimal Answer Approach (Benz).
- Evolutionary Game Theory
  - E.g. v. Rooij, Jäger
- Decision Theory
  - Relevance based approaches
  - E.g. A. Merin, R. v. Rooij

Game Theory

"A game is being played by a group of individuals whenever the fate of an individual in the group depends not only on his own actions but also on the actions of the rest of the group." (Binmore, 1990)

Game Theory and Pragmatics

In a very general sense we can say that we play a game together with other people whenever we have to decide between several actions such that the decision depends on:

- the choice of actions by others
- our preferences over the ultimate results.

Whether or not an utterance is successful depends on

- how it is taken up by its addressee
- the overall purpose of the current conversation.

Decision Theory

If a decision depends only on

- the state of the world,
- the actions to choose from and
- their outcomes

but not on

- the choice of actions by other agents,

then the problem belongs to decision theory.

Remark

The situation depicted in the graph for scalar implicatures is a problem for decision theory!

- **Decision theory:** decisions of individual agents
- **Game theory:** interdependent decisions of several agents.

Basic Issue

If Gricean Pragmatics can be modelled in:

- Decision Theory: Non-interactional view sufficient.
- Game Theory but not Decision Theory: Interactional view necessary!
    - H.H. Clark‘s Interactional Approach
    - Alignment Theory (Pickering, Garrod)
    - Conversational Analysis

PCIs and GCIs

- The goal is a foundational one.
- All implicatures will be treated as particularised conversational implicatures (PCIs).
- We will not discuss generalised conversational implicatures (GCIs) or Grice’ conventional implicatures.

Relevance Scale Approaches

Explanation of Implicatures

Relevance Scale Approaches (e.g. Merin, v. Rooij)

1. Read F +> ψ as: An utterance of F implicates that ψ.
2. The speaker chooses an answer A such that A is the most relevant proposition which S believes to be true.
3. Implicature F +> ψ is explained if it is known that S knows whether ψ and if $\neg\psi$ is more relevant than what the speaker said.

Relevant Gricean Maxims
(Short Form)

- Be truthful (Quality) and say as much as you can (Quantity) as long as it is relevant (Relevance).

Scalar Implicatures

(Quantity Implicature)

- Let $A(x)$ be a sentence frame.
- $\langle e_1, e_2, \ldots, e_n \rangle$ is a scale iff
  - $e_1, e_2, \ldots, e_n$ are elements of a closed lexical category.
  - for $i<j$: $A(e_i) \Rightarrow A(e_j)$ but $\neg A(e_j) \Rightarrow A(e_i)$.
- ➢ then for $i<j$: $A(e_j)$ +> $A(e_i)$
- Example: ⟨all, most, many, some⟩

Relevance Scale Approach

(Hirschberg, van Rooij; preliminary definition)

A theory about relevance implicatures is a **relevance scale approach** iff it defines or postulates a linear pre-order $\prec$ on propositions such that an utterance of proposition *A* implicates a proposition *H* iff *A* is less relevant than $\neg H$:

$$A \prec \neg H \Leftrightarrow A +\!> H$$

Relevance Scale Approach

- Let $\mathcal{M}$ be a set of propositions.
- Let $\leq$ be a linear well-founded pre-order on $\mathcal{M}$ with interpretation:

  $A \leq B \Leftrightarrow$ B is at least as relevant as A.

➢ then A +> $\neg$B iff $A < B$.

Relevance Scale Approach

(with real valued relevance measure)

- Let $\mathcal{M}$ be a set of propositions.
- $R : \mathcal{M} \rightarrow \Re$ real valued function with

$R(A) \leq R(B) \Leftrightarrow$ B is at least as relevant as A.

- then A +> $\neg$B iff $R(A) < R(B)$.

Examples

- Job Interview: J interviews E

  J: Do you speak Spanish?

  E: I speak some Portugese.

  +> E doesn’t speak Spanish.

- A in front of his obviously immobilised car.

  A: I am out of petrol.

  B: There is a garage around the corner. (G)

  +> The garage is open. (H)

The Italian Newspaper Example

Somewhere in the streets of Amsterdam...

a) J: Where can I buy an Italian newspaper?
b) E: At the station and at the Palace but nowhere else. (SE)
c) E: At the station. (A) / At the Palace. (B)

- With (Quantity) and (Quality):

  At the station (A) +> ¬ At the Palace (¬ B)

- A and A ∧ B are equally relevant, hence with (QQR):

  At the station (A) ~~+>~~ ¬ At the Palace (¬ B)

Two Types of Relevance Scale Approaches

- Argumentative view: Arthur Merin
- Non-Argumentative view: Robert van Rooij
  - Relevance Maximisation
  - Exhaustification

The Argumentative View

Arthur Merin (1999)

Information, relevance and social decision making

The Argumentative view

- Speaker tries to persuade the hearer of a hypothesis H.
- Hearer´s decision problem: Decide whether H or H* is true.
- Hearer´s expectations given by a probability space (Ω, P).

Example

If Eve has an interview for a job she wants to get, then

- her goal is to convince the interviewer that she is qualified for the job (H).
- Whatever she says is the more relevant the more it favours H and disfavours the opposite proposition $\bar{H}$.

Measuring the Update Potential of an Assertion A.

- Hearer's inclination to believe H prior to learning A:

$$P(H)/P(\bar{H})$$

- Inclination to believe H after learning A:

$$P^{+}(H)/P^{+}(\bar{H}) = P(H|A)/P(\bar{H}|A) =$$

$$= P(H)/P(\bar{H}) \times P(A|H)/P(A|\bar{H})$$

Using log (just a trick!) we get:

$\log P^+(H)/P^+(H^-) = \log P(H)/P(H^-) + \log P(A|H)/P(A|H^-)$

New = Old + update

$\log P(A|H)/P(A|H^-)$ can be seen as the **update potential** of proposition A with respect to H.

Relevance (Merin)

Intuitively: A proposition A is the more relevant to a hypothesis H the more it increases the inclination to believe H.

$$r_H(A) := \log P(A|H)/P(A|\bar{H})$$

- It is $r_{H^-}(A) = - r_H(A)$;
- If $r_H(A) = 0$, then A does not change the prior expectations about H.

An Example (Job interview)

v_1: Eve has ample of job experience and can take up a responsible position immediately.

v_2: Eve has done an internship and acquired there job relevant qualifications but needs some time to take over responsibility.

v_3: Eve has done an internship but acquired no relevant qualifications. She needs intensive training before she can start on the job.

v_4: Eve has just finished university without any work experience. Training is not an option.

- Interviewer's decision problem:
  - H: Employing Eve will be beneficial.
  - $\bar{H}$: Employing Eve will not be beneficial.
- All worlds equally probable.
- $H = \{v_1,v_2\}$, $\bar{H} = \{v_3,v_4\}$.
- Is $A = \{v_1,v_2,v_3\}$ positively relevant to H?

„I have work experience“

- Is $A = \{v_1, v_2, v_3\}$ positively relevant to H?

  „I have work experience“
- $r_H(A) = \log_2 P(A|H)/P(A|\bar{H}) =$

  $= \log_2 1/(1/2)$

  $= \log_2 2$

  $= 1 > 0$
- Hence A is positively relevant.

The Non-Argumentative View

Robert van Rooij (2003, 2004)

Quantity and quality of information exchange (2003)

Utility of mention-some questions (2004)

Assumptions

- The answering expert E tries to maximise the relevance of his answer.
- Relevance is defined by a real valued function R: $\wp(\Omega) \rightarrow \Re$.
- R only depends on the decision problem $((\Omega, P), A, u)$.
- E can only answer what he believes to be true.

We first provide an example which shows that we have to consider expected utilities when measuring the relevance of information.

The Job Interview Example

v_1: Eve has ample of job experience and can take up a responsible position immediately.

v_2: Eve has done an internship and acquired there job relevant qualifications but needs some time to take over responsibility.

v_3: Eve has done an internship but acquired no relevant qualifications. She needs intensive training before she can start on the job.

v_4: Eve has just finished university without any work experience. Training is not an option.

Adding Utilities

- Interviewer's decision problem:
  - a_1: Employ Eve.
  - a_2: Don't employ Eve.

u	v_1	v_2	v_3	v_4
a_1	10	1	-2	-5
a_2	0	0	0	0

All worlds equally probable

How to decide the decision problem?

Decision Criterion

- It is assumed that rational agents are **Bayesian utility maximisers**.
- If an agent chooses an action, then the action's expected utility must be maximal.

Expected Utility

Given a decision problem ((Ω, P),A,u), the expected utility of an action *a* is:

$$EU(a) = \sum_{v \in \Omega} P(v) \cdot u(v, a)$$

Effect of Learning $B = \{v_2, v_3\}$

Merin: $r_H(B) = 0$, hence B **irrelevant**!

$$EU(a_1) = \tfrac{1}{4} \times 10 + \tfrac{1}{4} \times 1 - \tfrac{1}{4} \times 2 - \tfrac{1}{4} \times 5$$

$$= \tfrac{1}{4} \times 4 = 1$$

$$EU(a_2) = 0 = EU(a_2|B)$$

$$EU(a_1|B) = \tfrac{1}{2} \times 1 - \tfrac{1}{2} \times 2 = -\tfrac{1}{2}$$

Negatively relevant !

Sample Value of Information

(Measures of Relevance I)

New information A is relevant if

- it leads to a different choice of action, and
- it is the more relevant the more it increases thereby expected utility.

Sample Value of Information

- Let ((Ω, P),A,u) be a given decision problem.
- Let a* be the action with maximal expected utility before learning *A*.

Possible definition of **Relevance** of *A*:

$$UV(A) = \max_{a \in \mathcal{A}} EU(a|A) - EU(a^*|A).$$

(Sample Value of Information)

Utility Value
(Measures of Relevance II)

Possible alternative e.g.:

New information A is relevant if

- it increases expected utility.
- it is the more relevant the more it increases it.

$$UV'(A) = \max_{a \in \mathcal{A}} EU(a|A) - \max_{a \in \mathcal{A}} EU(a).$$

The Italian Newspaper Example

Somewhere in the streets of Amsterdam...

a) J: Where can I buy an Italian newspaper?
b) E: At the station and at the Palace but nowhere else. (SE)
c) E: At the station. (A) / At the Palace. (B)

Possible Worlds

	Station	Palace
w_1	+	+
w_2	+	-
w_3	-	+
w_4	-	-

Answers:

- A: at the station ($A = \{w_1, w_2\}$)
- B: at the Palace ($B = \{w_1, w_3\}$)

Actions and Answers

- I’s actions:
  - a: going to station;
  - b: going to Palace;
- Let utilities be such that they only distinguish between success (value 1) and failure (value 0).

Scenario I

If:

1. $P_I(A) = P_I(B)$
2. *E* knows that $A \wedge B$, i.e. $P_E(A \cap B) = 1$.

Then:

- With both, sample value and utility value, all three answers A, B, SE are equally relevant.

Scenario II

If:

1. $P_I(A) > P_I(B)$
2. *E* knows that $A \wedge B$, i.e. $P_E(A \cap B)=1$.

Then:

- With sample value of information: Only B is relevant.
- With utility value: A, B, and $A \wedge B$ are equally relevant.

Scenario III

If:

1. $P_I(A) > P_I(B)$
2. *E* knows only that $\neg A$, i.e. $P_E(\neg A)=1$.

Then

- With sample value of information: $\neg A$ is relevant.
- With utility value: the uninformative answer is the most relevant answer.

- **Needed:** Uniform definition of relevance that explains all examples.
- **But:** We will see in the last lesson that there are principled examples that cannot be explained by any approach based on maximisation of relevance.