# Game Theory and Gricean Pragmatics Lesson III

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# Course Overview

- Lesson 1: Introduction
  - From Grice to Lewis
  - Relevance Scale Approaches
- Lesson 2: Signalling Games
  - Lewis‘ Signalling Conventions
  - Parikh‘s Radical Underspecification Model
- Lesson 3: The Optimal Answer Approach I
- Lesson 4: The Optimal Answer Approach II
  - Comparison with Relevance Scale Approaches
  - Decision Contexts with Multiple Objectives

# Optimal Answer Approach

Lesson III – April, 5th

# Overview of Lesson III

- Natural Information and Conversational Implicatures
  - An Example: Scalar Implicatures
  - Natural Information and Conversational Implicatures
  - Calculating Implicatures in Signalling Games
- Optimal Answers
  - Core Examples
  - Optimal Answers in Support Problems
  - Examples
- Support Problems and Signalling Games

# The Agenda

Putting Grice on Lewisean feet!

# Natural Information and Conversational Implicatures

# Explanation of Implicatures Optimal Answer Approach

1. Start with a signalling game where the hearer interprets forms by their literal meaning.
2. Impose pragmatic constraints and calculate equilibria that solve this game.
3. Implicature F +> φ is explained if for all solutions (S,H):

$$S^{-1}(F) \models \varphi$$

# Contrast

In an information based approach:

- Implicatures emerge from **indicated meaning** (in the sense of Lewis).
- Implicatures are **not** initial candidate interpretations.
- Speaker does **not** maximise relevance.
- No diachronic process.

- Assumption: speaker and hearer use language according to a semantic convention.
- Goal: Explain how implicatures can emerge out of semantic language use.

> Non-reductionist perspective.

# Representation of Assumption

- Semantics defines interpretation of forms.
- Let [F] denote the semantic meaning.
- Hence, assumption: $H(F)=[F]$, i.e.:

  $H(F)$ is the semantic meaning of $F$

- Semantic meaning $\cong$ Lewis' imperative signal.

# Background (Repetition)

Lewis (IV.4,1996) distinguishes between

- indicative signals
- imperative signals

Two possible definitions of **meaning**:

- **Indicative:**

$$[F] = M \text{ :iff } S^{-1}(F)=M$$

- **Imperative:**

$$[F] = M \text{ :iff } H(F)=M$$

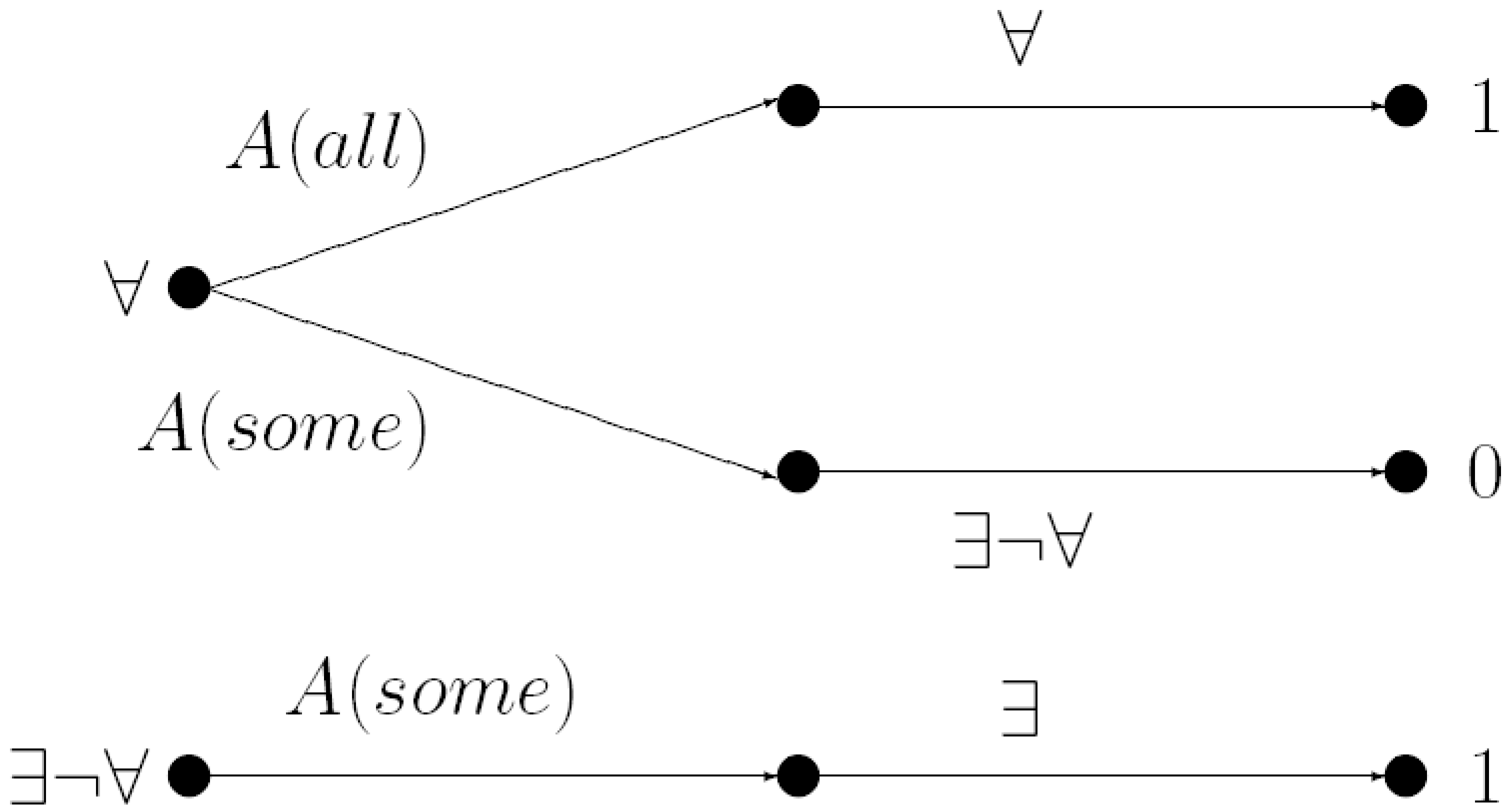
# An Example

We consider the standard example:

Some of the boys came to the party.

- **said:** at least two came
- **implicated:** not all came

# The Game



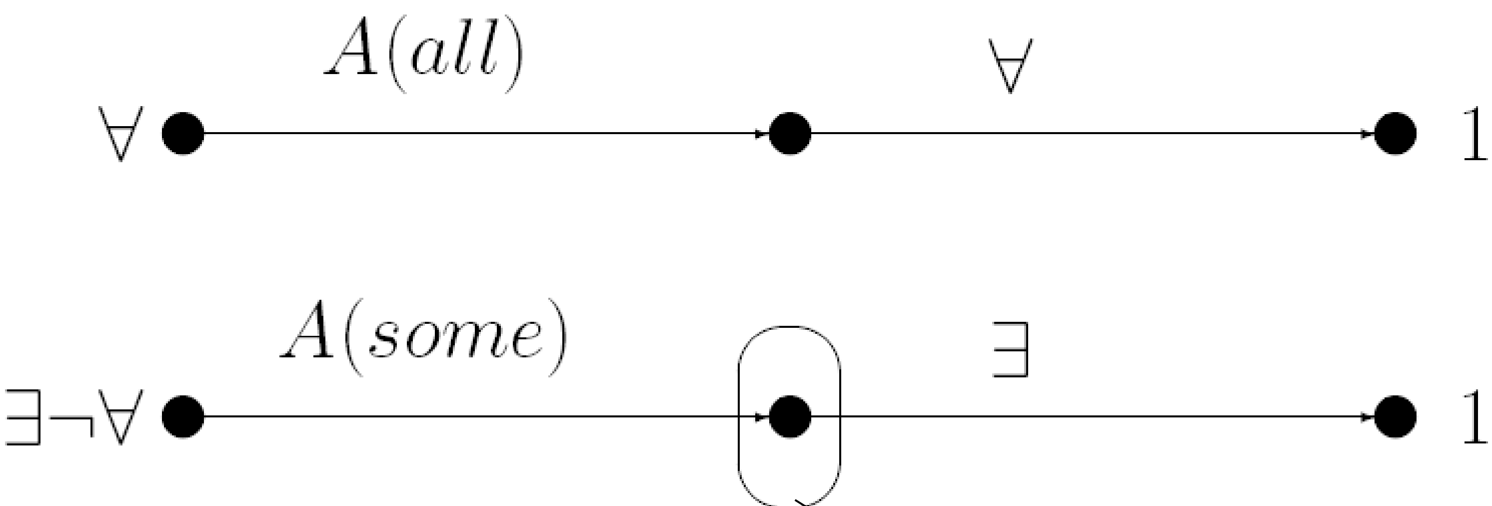
$\forall$
$A(all)$
$1$
$\forall$
$A(some)$
$0$
$\exists\neg\forall$
$A(some)$
$\exists$
$\exists\neg\forall$
$1$

# The Solved Game

$$\begin{tikzcd} \forall\ \bullet \arrow[r, "A(all)"] & \bullet \arrow[r, "\forall"] & \bullet\ 1 \end{tikzcd}$$

$$\begin{tikzcd} \exists\neg\forall\ \bullet \arrow[r, "A(some)"] & \bullet \arrow[r, "\exists"] & \bullet\ 1 \end{tikzcd}$$

# The hearer can infer after receiving A(some) that:




In all branches that contain “some,” it is the case that some but not all boys came.

# Natural Information and Conversational Implicatures

# Natural and Non-Natural Meaning

Grice distinguished between

- natural meaning
- non-natural meaning

➢ Communicated meaning is non-natural meaning.

# Example

1. I show Mr. X a photograph of Mr. Y displaying undue familiarity to Mrs. X.
2. I draw a picture of Mr. Y behaving in this manner and show it to Mr. X.

- The photograph **naturally** means that Mr. Y was unduly familiar to Mrs. X
- The picture **non-naturally** means that Mr. Y was unduly familiar to Mrs. X

- Taking a photo of a scene necessarily entails that the scene is real.
  - Every branch which contains a showing of a photo must contain a situation which is depicted by it.
  - The showing of the photo means naturally that there was a situation where Mr. Y was unduly familiar with Mrs. X.
- The drawing of a picture does not imply that the depicted scene is real.

# Natural Information of Signals

- Let **G** be a signalling game.
- Let **S** be a set of strategy pairs (S,H).
- We identify the natural information of a form F in **G** with respect to **S** with:

**The set of all branches of G where the speaker chooses** F.

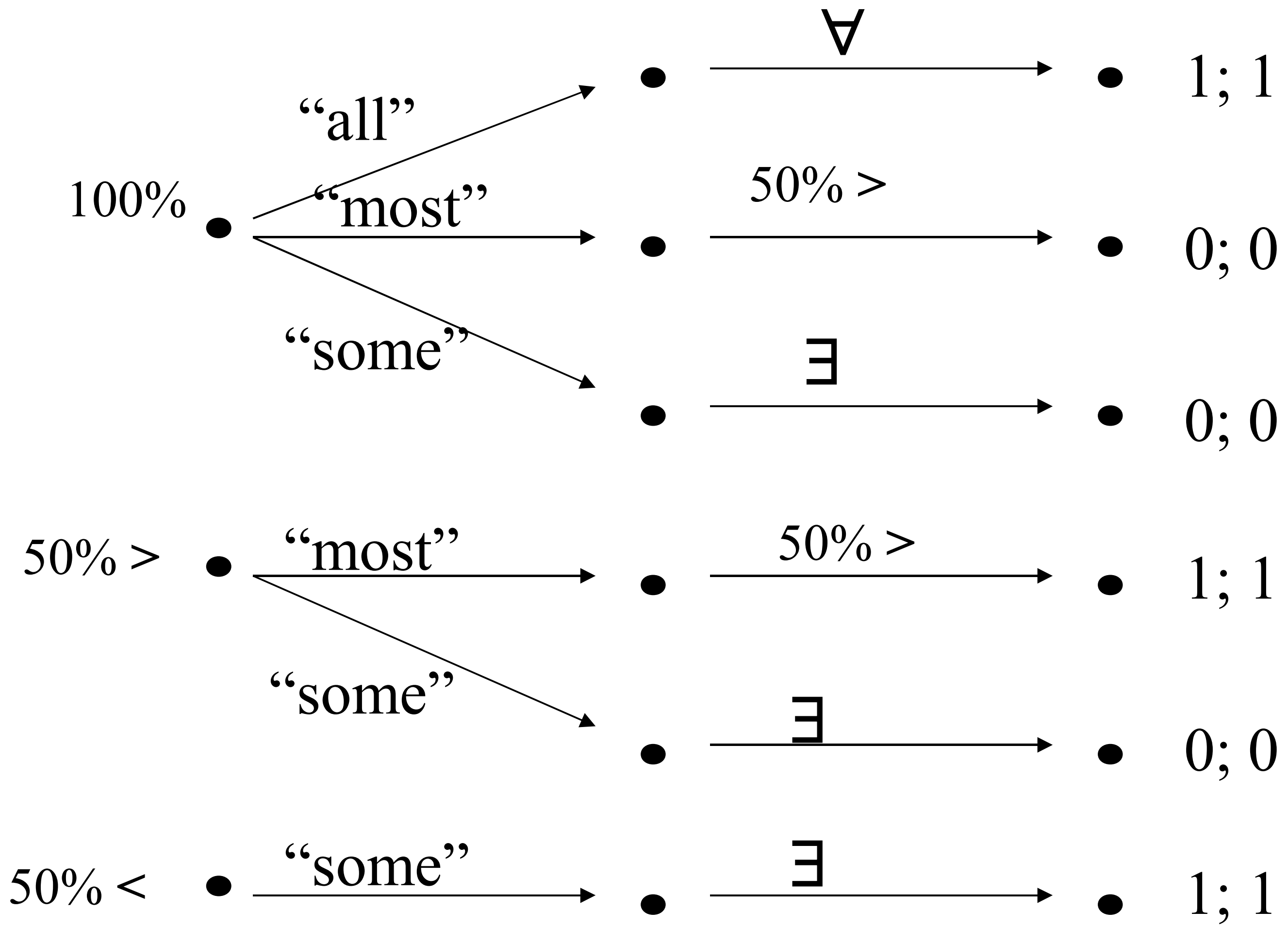
- Information coincides with $S^{-1}(F)$ in case of simple Lewisean signalling games.
- Generalises to arbitrary games which contain semantic interpretation games in embedded form.
- Conversational Implicatures are implied by the natural information of an utterance.

# The Standard Example reconsidered

Some of the boys came to the party.

- **said:** at least two came
- **implicated:** not all came

# The game defined by pure semantics



# The game after optimising speaker’s strategy

100% • $\xrightarrow{\text{“all”}}$ • $\xrightarrow{\forall}$ • 2; 2

50% > • $\xrightarrow{\text{“most”}}$ • $\xrightarrow{50\% >}$ • 1; 1

50% < • $\xrightarrow{\text{“some”}}$ • $\xrightarrow{\exists}$ • 1; 1

In all branches that contain “some,” the initial situation is “50% < ”

# The possible worlds

- $w_1$: 100% of the boys came to the party.
- $w_2$: More than 50% of the boys came to the party.
- $w_3$: Less than 50% of the boys came to the party.

# The possible Branches of the Game Tree

$$\begin{aligned}
&\langle w_1, A(all), \{w_1\}\rangle,\\
&\langle w_1, A(most), \{w_1, w_2\}\rangle,\\
&\langle w_1, A(some), \{w_1, w_2, w_3\}\rangle,\\
&\langle w_2, A(most), \{w_1, w_2\}\rangle,\\
&\langle w_2, A(some), \{w_1, w_2, w_3\}\rangle,\\
&\langle w_3, A(some), \{w_1, w_2, w_3\}\rangle.
\end{aligned}$$

# The unique signalling strategy that solves this game:

| $S$ | $100\%$ | $>50\%$ | $\leq 50\%$ |
|---|---|---|---|
| | $A(all)$ | $A(most)$ | $A(some)$ |

# The Natural Information carried by utterance A(some)

- The branches allowed by strategy S:

  $\langle w_1, A(\text{all}), \{w_1\}\rangle$

  $\langle w_2, A(\text{most}), \{w_1, w_2\}\rangle$

  $\langle w_3, A(\text{some}), \{w_1, w_2, w_3\}\rangle$

- Natural information carried by A(some):

  $\{\langle w_3, A(\text{some}), \{w_1, w_2, w_3\}\rangle\}$

Hence: An utterance of A(some) is a true sign that less than 50% came to the party.

# Implicatures in Signalling Games

A special case

# As Signalling Game (Repetition)

A signalling game is a tuple:

$$\langle N, \Theta, p, (A_1, A_2), (u_1, u_2) \rangle$$

- N: Set of two players S,H.
- Θ: Set of types representing the speakers private information.
- p: A probability measure over Θ representing the hearer's expectations about the speaker's type.

- $(A_1, A_2)$: the speaker's and hearer's action sets:
  - $A_1$ is a set of forms **F** / meanings **M**.
  - $A_2$ is a set of actions.
- $(u_1, u_2)$: the speaker's and hearer's payoff functions with

$$u_i: A_1 \times A_2 \times \Theta \rightarrow \mathbf{R}$$

# Strategies in a Signalling Game

- Let $[\ ] : \mathbf{F} \rightarrow \mathbf{M}$ be a given semantics.
- The speaker's strategies are of the form:

  $S : \Theta \rightarrow A_1$ such that

  $S(\theta) = F \Rightarrow \theta \subseteq [F]$

  i.e. if the speaker says F, then he knows that F is true (**Maxim of Quality**).

# Definition of Implicature
(special case)

Given a signalling game as before, then an implicature

$$F +> \psi$$

is explained iff the following set is a subset of $[\psi] = \{w \in \Omega | w \models \psi\}$:

$$\boxed{\{w \in \Omega | \exists \text{ solution } (S, H) \exists \theta \in \Theta : w \in \theta \wedge S(\theta) = F\}}$$

# Preview

1. Later, we apply this criterion to calculating implicatures of answers.
2. The definition depends on the method of finding solutions.

- First we need a method for calculating **optimal answers**.
- The resulting signalling and interpretation strategies are then the solutions which we use as imput for calculating implicatures.

# Optimal Answers

# Core Examples

# Italian Newspaper

Somewhere in the streets of Amsterdam...

a) J: Where can I buy an Italian newspaper?
b) E: At the station and at the Palace but nowhere else. (SE)
c) E: At the station. (A) / At the Palace. (B)

- The answer (SE) is called **strongly exhaustive**.
- The answers (A) and (B) are called **mention–some answers**.

➢ A and B are as good as SE or as $A \wedge \neg B$:

a) E: There are Italian newspapers at the station but none at the Palace.

# Partial Answers

If E knows only that ¬A, then ¬A is an optimal answer:

a) E: There are no Italian newspapers at the station.

If E only knows that the Palace sells foreign newspapers, then this is an optimal answer:

a) E: The Palace has foreign newspapers.

Partial answers may also arise in situations where speaker E has full knowledge:

- I: I need patrol for my car. Where can I get it?
  E: There is a garage round the corner.

- J: Where can I buy an Italian newspaper?
  E: There is a news shop round the corner.

# Optimal Answers in Support Problems

The Framework

# Support Problem

**Definition:** A support problem is a five–tuple $(\Omega, P_E, P_I, \mathcal{A}, u)$ such that

1. $(\Omega, P_E)$ and $(\Omega, P_I)$ are finite probability spaces,
2. $(\Omega, P_I, \mathcal{A}, u)$ is a decision problem.

Let $K := \{w \in \Omega \mid P_E(w) > 0\}$ (E‘s knowledge set).

Then, we assume in addition:

1. for all $A \subseteq \Omega$: $P_E(A) = P_I(A|K)$

# Support Problem

$$
\begin{array}{ccccc}
\text{Expert E} & & \text{I decides} & & \text{Evaluation} \\
\text{answers} & & \text{for action} & & \\
\downarrow & & \downarrow & & \downarrow \\
\bullet & \xrightarrow{A} & \bullet & \xrightarrow{a} & \bullet \\
\uparrow & & \uparrow & & \uparrow \\
\text{expectations} & & \text{expectations} & & \text{utility} \\
\text{of E} & & \text{of I} & & \text{function} \\
(\Omega, P_E) & & (\Omega, P_I) & & u(v, a)
\end{array}
$$

# I's Decision Situation

*I* optimises expected utilities of actions:

$$EU(a) = \sum_{v \in \Omega} P(v) \times u(v, a).$$

After learning A, *I* has to optimise:

$$EU(a, A) = \sum_{v \in \Omega} P(v|A) \times u(v, a).$$

- I will choose an action $a_A$ that optimises expected utility, i.e. for all actions $b$

$$EU(b,A) \leq EU(a_A,A)$$

Given answer $A$, H($A$) = $a_A$.

- For simplicity we assume that I's choice $a_A$ is commonly known.

# E's Decision Situation

E optimises expected utilities of answers:

$$EU_E(A) := \sum_{v \in \Omega} P_E(v) \times u(v, a_A).$$

- (Quality): The speaker can only say what he thinks to be true.
- (Quality) restricts answers to:

$$Adm_S := \{A \subseteq \Omega \mid P_E(A) = 1\}$$

- Hence, E will choose his answers from:

$$\mathrm{Op}_{\mathcal{S}} = \{A \in Adm_S | \forall B \in Adm_S \, EU_E(B) \leq EU_E(A)\}.$$

# Examples

The Italian Newspaper Examples

# Italian Newspaper

Somewhere in the streets of Amsterdam...

a) J: Where can I buy an Italian newspaper?
b) E: At the station and at the Palace but nowhere else. (SE)
c) E: At the station. (A) / At the Palace. (B)

# Possible Worlds (equally probable)

| | Station | Palace |
|---|---|---|
| $w_1$ | + | + |
| $w_2$ | + | - |
| $w_3$ | - | + |
| $w_4$ | - | - |

# Actions and Answers

- I’s actions:
  - a: going to station;
  - b: going to Palace;
- Answers:
  - A: at the station ($A = \{w_1, w_2\}$)
  - B: at the Palace ($B = \{w_1, w_3\}$)

- Let utilities be such that they only distinguish between success (value 1) and failure (value 0).
- Let's consider answer $A = \{w_1, w_2\}$.
- Assume that the speaker knows that A, i.e. there are Italian newspapers at the station.

# The Calculation

- If hearing $A$ induces hearer to choose $a$ (i.e. $a_A=a$ 'going to station'):

$$EU_E(A) = \sum_{v \in \Omega} P_E(v) \times u(v, a_A) = \sum_{v \in A} P_E(v) \times u(v, a) = 1$$

- If hearing $A$ induces hearer to choose $b$ (i.e. $a_A=b$ 'going to Palace'):
  - If $P_E(B) = 1$, then $EU_E(A) = EU_E(B) = 1$.
  - $P_E(B) < 1$ leads to a contradiction.

$P_E(B) < 1$ leads to a contradiction:

1. $a_A = b$ implies $EU_I(b|A) \geq EU_I(a|A) = 1$.
2. Hence, $EU_I(b|A) = \sum_{v\in A} P_I(v)\, u(v,b) = 1$.
3. Therefore $P_I(B|A) = 1$, hence $P_I(B \cap A) = P_I(A)$, hence $P_I(A \setminus B) = 0$.
4. $P_E(A \setminus B) = 0$, because $\exists K$: $P_E(X) = P_I(X|K)$.
5. $P_E(B \cap A) = P_E(A) = 1$, hence $P_E(B) = 1$.

# Case: Speaker knows that Italian newspaper are at both places

- Calculation showed that $EU_E(A) = 1$.
- Expected utility cannot be higher than 1 (due to assumptions).
- Similar: $EU_E(B) = 1$; $EU_E(A \wedge B) = 1$.
- Hence, all these answers are equally optimal.

# More Cases

- E knows that $A$ and $B$:

  $EU_E(A) = EU_E(B) = EU_E(A \wedge B)$
- E knows that $A$ and $\neg B$:

  $EU_E(A) = EU_E(A \wedge \neg B)$
- E knows only that $A$:

  For all admissible $C$: $EU_E(C) \leq EU_E(A)$

The following example shows how the method of finding optimal answers in support problems interacts with the general theory of implicatures in signalling games.

# Hip Hop at Roter Salon

John loves to dance to Salsa music and he loves to dance to Hip Hop but he can’t stand it if a club mixes both styles.

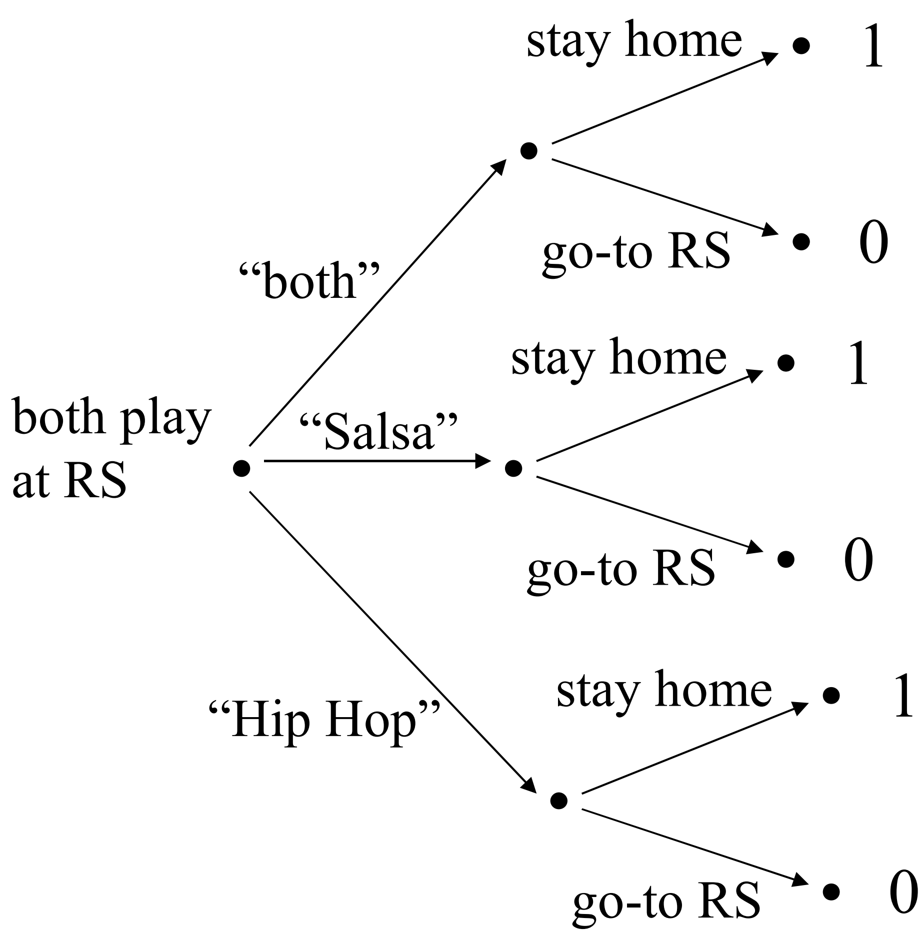
J: I want to dance tonight. Is the Music in Roter Salon ok?

E: Tonight they play Hip Hop at the Roter Salon.

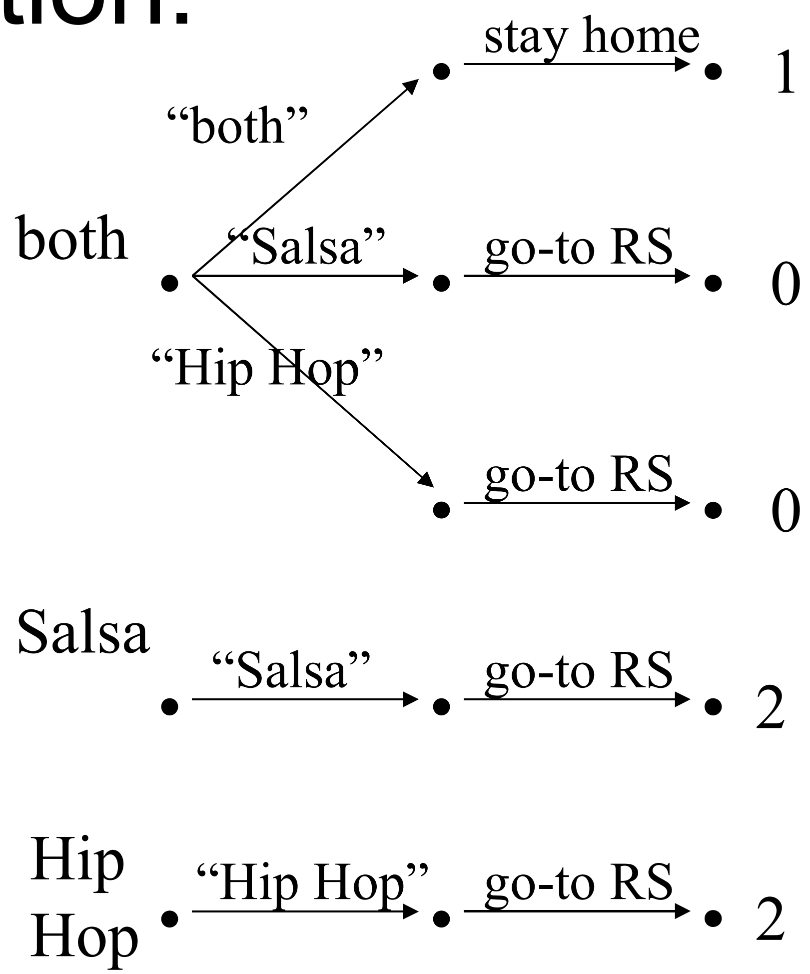
+> They play only Hip Hop.

# A game tree for the situation where both Salsa and Hip Hop are playing

RS = Roter Salon



# After the first step of backward induction:



# After the second step of backward induction:

both $\bullet \xrightarrow{\text{“both”}} \bullet \xrightarrow{\text{stay home}} \bullet$ 1

Salsa $\bullet \xrightarrow{\text{“Salsa”}} \bullet \xrightarrow{\text{go-to RS}} \bullet$ 2

Hip Hop $\bullet \xrightarrow{\text{“Hip Hop”}} \bullet \xrightarrow{\text{go-to RS}} \bullet$ 2

In all branches that contain “Salsa” the initial situation is such that only Salsa is playing at the Roter Salon.

Hence: “Salsa” **implicates** that only Salsa is playing at Roter Salon

If we say that a proposition is the more relevant the higher the expected utility after learning it, then relevance scale approaches predict that „Hip Hop“ implicates that both, Salsa and Hip Hop, are playing.

Worst case compatible with what was said!

# Hip Hop at Roter Salon

Abbreviations:

$H(x)$: There is Hip Hop at $x$;

$S(x)$: There is Salsa at $x$.

- Good(x) := $(H(x) \vee S(x)) \wedge \neg(H(x) \wedge S(x))$

# Assumptions

I. Equal Probabilities

$$\exists h > 0 \, \forall x \, P_I(H(x)) = h;$$

$$\exists s > 0 \, \forall x \, P_I(S(x)) = s;$$

$$\exists g > 0 \, \forall x \, P_I(Good(x)) = g.$$

I. Independence: $X, Y \in \{H, S, Good\}$

$$a \neq b \Rightarrow P_I(X(a) \wedge Y(b)) = P_I(X(a)) \times P_I(Y(b)).$$

I. Learning $H(x)$ or $S(x)$ raises expected utility of going to salon $x$:

a) $EU_I(\text{going-to-}x) < EU_I(\text{stay-home}) < EU_I(\text{going-to-}x|H(x))$

b) $EU_I(\text{going-to-}x) < EU_I(\text{stay-home}) < EU_I(\text{going-to-}x|S(x))$

# Violating Assumptions II

The Roter Salon and the Grüner Salon share two DJs. One of them only plays Salsa, the other one mainly plays Hip Hop but mixes into it some Salsa. There are only these two Djs, and if one of them is at the Roter Salon, then the other one is at the Grüner Salon. John loves to dance to Salsa music and he loves to dance to Hip Hop but he can’t stand it if a club mixes both styles.

J: I want to dance tonight. Is the Music in Roter Salon ok?

E: Tonight they play Hip Hop at the Roter Salon.

# Support Problems and Signalling Games

In our model, the speaker finds an optimal answer by backward induction in support problems.

This is not a standard method for solving coordination problems in signalling games.

# Signalling Game

A signalling game is a tuple:

$$\langle N,\Theta, p, (A_1,A_2), (u_1, u_2)\rangle$$

- N: Set of two players S,H.
- Θ: Set of types representing the speakers private information
- p: A probability measure over Θ representing the hearer's expectations about S' type.

# Solution to a Signalling Game

- The standard solution concept for Signalling games is that of a **perfect Bayesian equilibrium**!
- (S,H) strategies:

$$S : \Theta \rightarrow A_1$$

$$H : A_1 \rightarrow A_2$$

# Perfect Bayesian equilibrium (S,H)

- $\forall\theta\ S(\theta) \in \text{argmax}_F\ u_1(F,H(F),\theta)$
- $\forall F\ H(F) \in \text{argmax}_M \sum_\theta \mu(\theta|F) \times u_2(F,M,\theta)$

where $\mu$ is defined by

- $\mu(\theta|F) = 0$ if $S(\theta) \neq F$
- $\mu(\theta|F) = p(\theta) / p(S^{-1}[F])$ if $S(\theta)=F$

if $p(S^{-1}[F]) > 0$, else $\mu(\theta|F)$ is arbitrary.

# Task

Given:

- a set of support problems **S** with fixed decision problem $(\Omega, P_I, A, u)$ for a

Wanted:

- Representation as signalling game:

$$\langle N, \Theta, p, (A_E, A_I), (u_E, u_I) \rangle$$

# Construction

- Let $\sigma=(\Omega,P_E,P_I,A,u)$ be a given support problem.
- Remember: there is a common prior P on $\Omega$ such that:

  $P_E(X) = P_I(X|K_\sigma)$ for $K_\sigma := \{w\in \Omega | P_E(w) > 0\}$

➢ Add $K_\sigma$ to $\Theta$ (i.e. $\Theta = \{K_\sigma | \sigma\in \mathcal{S}\}$)

- The speaker's action set $A_E$ is identical with a set of forms ***F*** / meanings ***M****.*
- The hearer's action set is identical to the action set of $\sigma$.

1. The game is a game of pure coordination with respect to joint payoff functions

$$u_i: \boldsymbol{F} \times A_I \times \Theta \rightarrow \mathbf{R}$$

- $u_I(A,a,K) := EU_I(a|K)$
- $u_E(A,a,K) := EU_E(a|K) \quad (= EU_I(a|K))$

1. p is arbitrary (as long as $p(\theta)>0$ for $\theta \in \Theta$).
1. Forms F have to be interpreted by their semantic meaning [F].
2. The speaker has to conform to the **maxim of quality**, i.e. $S(K_\sigma) \in Adm_\sigma$

# Result

The strategy pairs defined by:

$$S(K_\sigma) \in Op_\sigma,\ H(A) = a_A$$

- are Perfect Bayesian Equilibria of the associated signalling game.
- they (weakly) Pareto dominate all other strategy pairs (S‘,H‘).