Signalling Games and Pragmatics
Day IV

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The Course

- Day I: Introduction: From Grice to Lewis
- Day II: Basics of Game and Decision Theory
- Day III: Two Theories of Implicatures (Parikh, Jäger)
- **Day IV: Best Answer Approach**
- Day V: Utility and Relevance

Best Answer Approach

Day 4 – August, 10th

Overview

- An Information Based Approach
  - An Example: Scalar Implicatures
  - Natural Information and Conversational Implicatures
  - Calculating Implicatures in Signalling Games
- Optimal Answers
  - Core Examples
  - The Framework
  - Examples
- Implicatures of Answers

An Information Based Approach

Lewisising Grice

Game and Decision Theoretic Approaches to Gricean Pragmatics

Distinguish between Approaches based on:

- Classical Game Theory
  - Underspecification based Approach (P. Parikh).
  - Information Based Approach (Benz).
- Evolutionary Game Theory
  - E.g. v. Rooij, Jäger
- Decision Theory
  - Relevance base approaches
  - E.g. Merin, v. Rooij

Explanation of Implicatures

Disambiguation based Approach (e.g. Parikh)

1. Start with a signalling game $\mathcal{G}$ which allows many candidate interpretations for critical forms.
2. Impose pragmatic constraints and calculate equilibria that solve this game.
3. Implicature F +> ψ is explained if it holds for the solution (S,H):

$$H(F) \models \psi$$

Explanation of Implicatures

Diachronic Approach (e.g. Jäger)

1. Start with a signalling game $\mathcal{G}$ and a first strategy pair (S,H).
2. Diachronically, a stable strategy pair (S',H') will evolve from (S,H).
3. Implicature F +> ψ is explained if

$$H'(F) \models \psi$$

Explanation of Implicatures

Information based approach

1. Start with a signalling game where the hearer interprets forms by their literal meaning.
2. Impose pragmatic constraints and calculate equilibria that solve this game.
3. Implicature F +> φ is explained if for all solutions (S,H):

$$S^{-1}(F) \models \varphi$$

Background

Lewis (IV.4,1996) distinguishes between

- indicative signals
- imperative signals

Two possible definitions of **meaning**:

- **Indicative:**

$$[F] = M :\text{iff } S^{-1}(F)=M$$

- **Imperative:**

$$[F] = M :\text{iff } H(F)=M$$

Contrast

In an information based approach:

- Implicatures emerge from **indicated meaning** (in the sense of Lewis).
- Implicatures are **not** initial candidate interpretations.

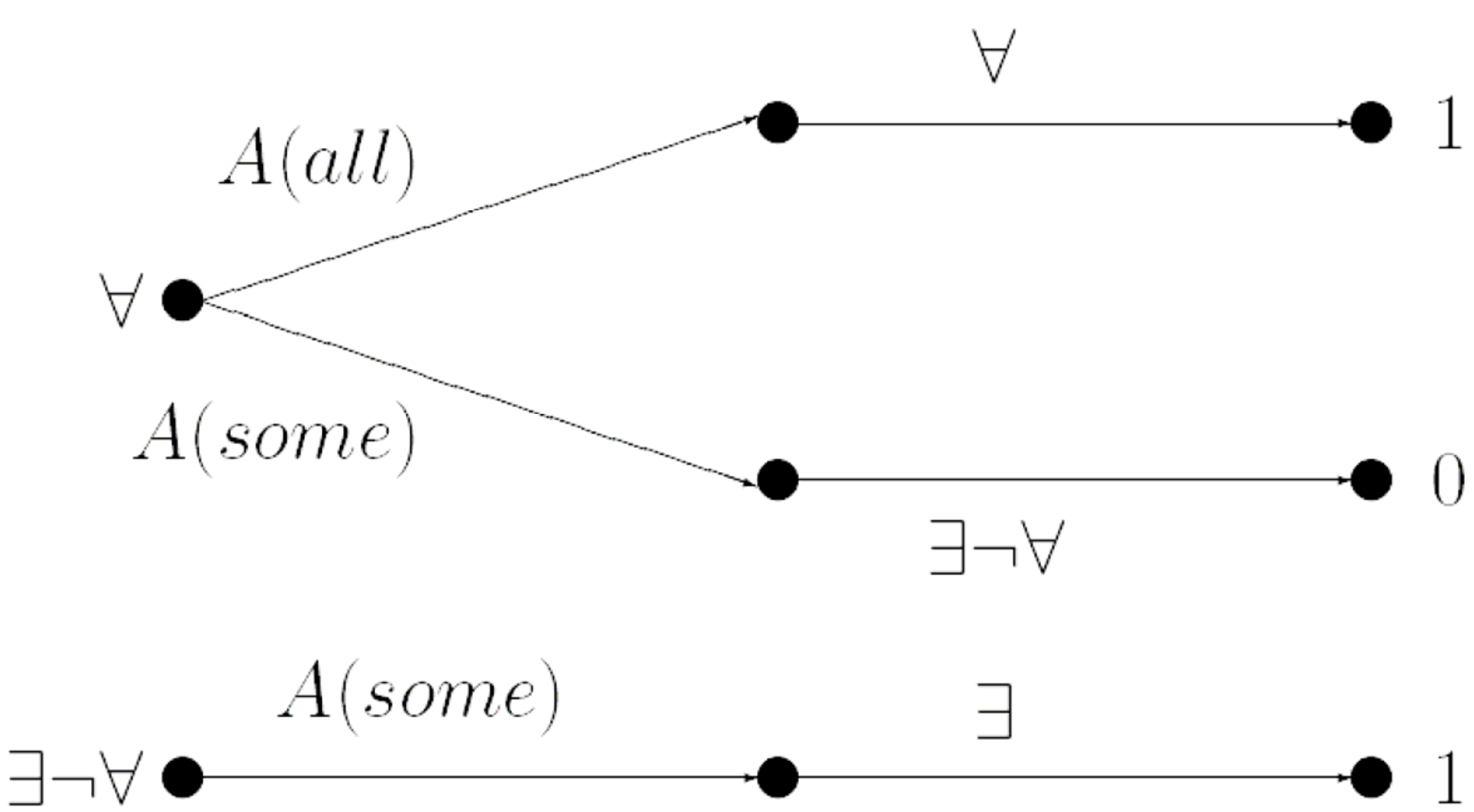
An Example

We consider the standard example:

Some of the boys came to the party.

- **said:** at least two came
- **implicated:** not all came

The Game

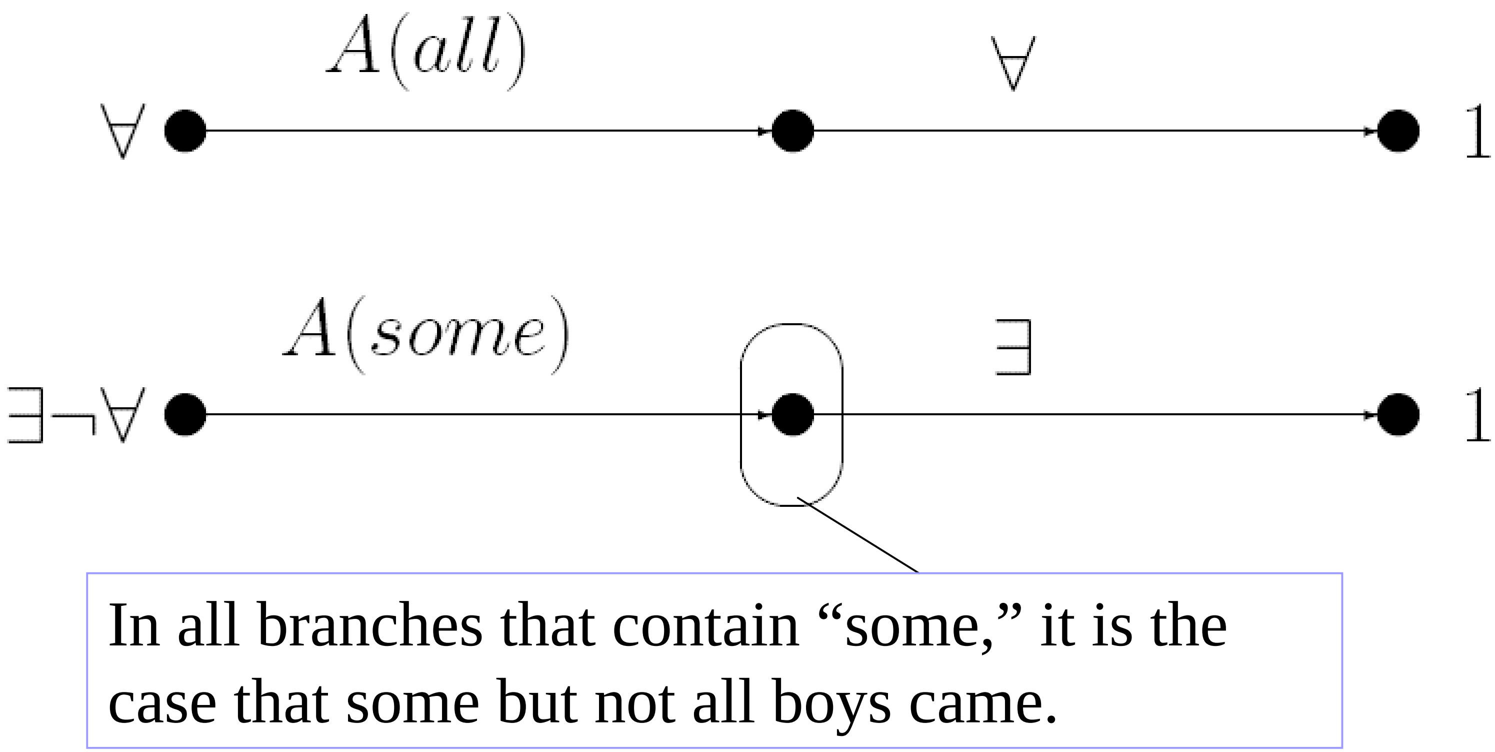


The Solved Game

$$\forall \; \bullet \xrightarrow{A(all)} \bullet \xrightarrow{\forall} \bullet \; 1$$

$$\exists\neg\forall \; \bullet \xrightarrow{A(some)} \bullet \xrightarrow{\exists} \bullet \; 1$$

The hearer can infer after receiving A(some) that:



Standard Explanation based on Maxims (from Day I)

Let $A(x) \equiv$ “x of the boys came to the party”

1. The speaker had the choice between the forms A(all) and A(some).
2. A(all) is more informative than A(some) and the additional information is also relevant.
3. Hence, if all of the boys came, then A(all) is preferred over A(some) (Quantity) + (Relevance).

4. The speaker said A(some).
5. Hence it cannot be the case that all came.
6. Therefore some but not all came to the party.

Natural Information and Conversational Implicatures

Natural and Non-Natural Meaning

Grice distinguished between

- natural meaning
- non-natural meaning
- Communicated meaning is non-natural meaning.

Example

1. I show Mr. X a photograph of Mr. Y displaying undue familiarity to Mrs. X.
2. I draw a picture of Mr. Y behaving in this manner and show it to Mr. X.

- The photograph **naturally** means that Mr. Y was unduly familiar to Mrs. X
- The picture **non-naturally** means that Mr. Y was unduly familiar to Mrs. X

- Taking a photo of a scene necessarily entails that the scene is real.
  - Every branch which contains a showing of a photo must contain a situation which is depicted by it.
  - The showing of the photo means naturally that there was a situation where Mr. Y was unduly familiar with Mrs. X.
- The drawing of a picture does not imply that the depicted scene is real.

Natural Information of Signals

- Let $\mathcal{G}$ be a signalling game.
- Let $\mathcal{S}$ be a set of strategy pairs (S,H).
- We identify the natural information of a form F in $\mathcal{G}$ with respect to $\mathcal{S}$ with:

The set of all branches of $\mathcal{G}$ where the speaker chooses F.

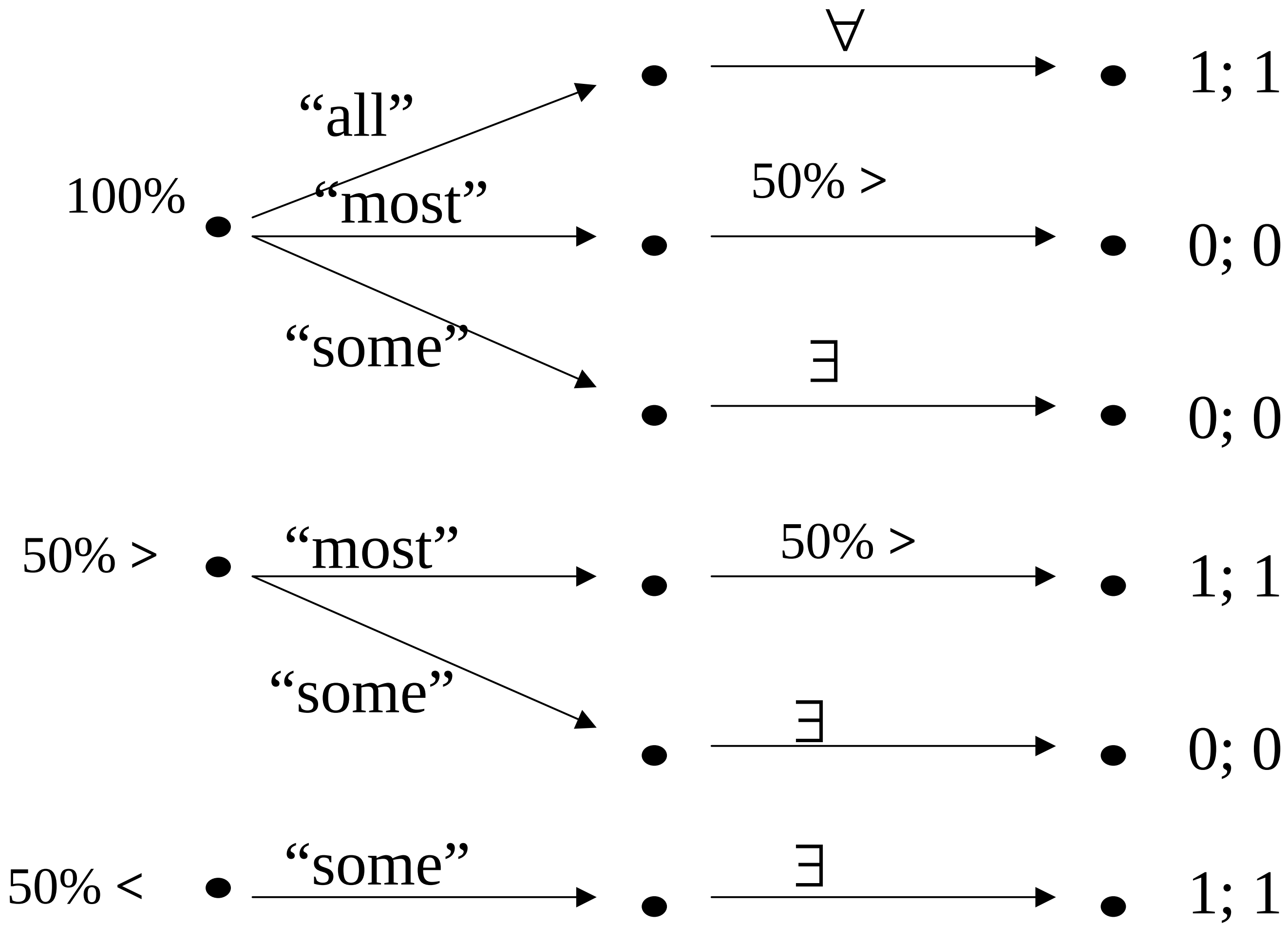
- Information coincides with $S^{-1}(F)$ in case of simple Lewisean signalling games.
- Generalises to arbitrary games which contain semantic interpretation games in embedded form.
- Conversational Implicatures are implied by the natural information of an utterance.

Scalar Implicatures Reconsidered

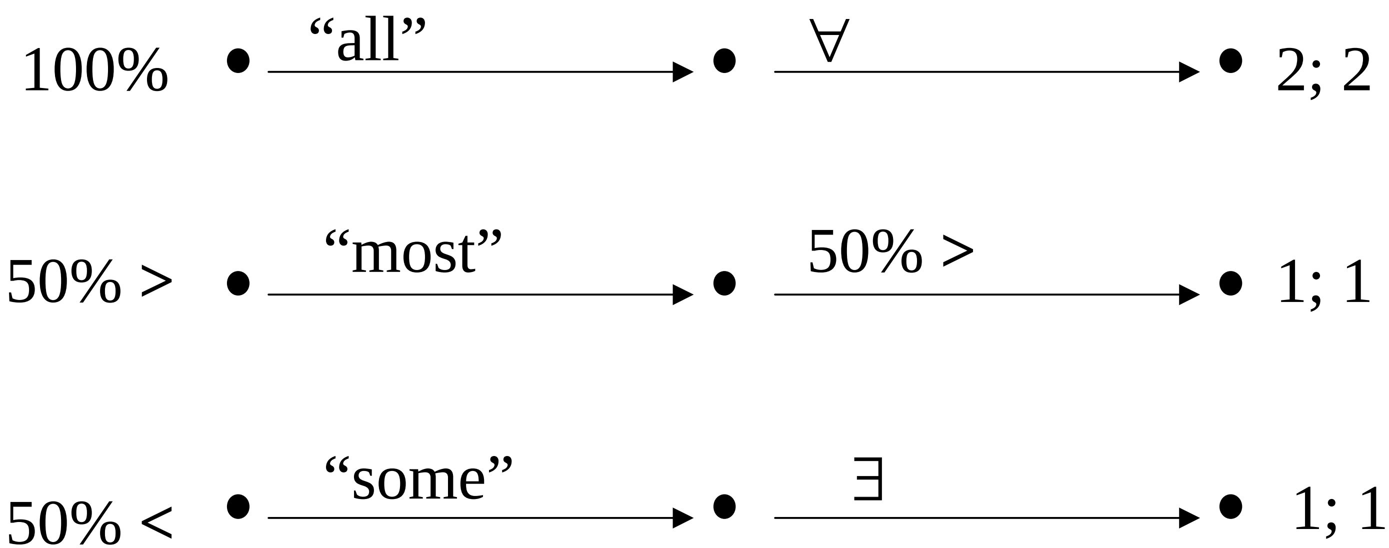
Some of the boys came to the party.

- **said:** at least two came
- **implicated:** not all came

The game defined by pure semantics



The game after optimising speaker's strategy



In all branches that contain "some," the initial situation is "50% < "

The possible worlds

- w_1: 100% of the boys came to the party.
- w_2: More than 50% of the boys came to the party.
- w_3: Less than 50% of the boys came to the party.

The possible Branches of the Game Tree

$$
\begin{aligned}
&\langle w_1, A(all), \{w_1\}\rangle,\\
&\langle w_1, A(most), \{w_1, w_2\}\rangle,\\
&\langle w_1, A(some), \{w_1, w_2, w_3\}\rangle,\\
&\langle w_2, A(most), \{w_1, w_2\}\rangle,\\
&\langle w_2, A(some), \{w_1, w_2, w_3\}\rangle,\\
&\langle w_3, A(some), \{w_1, w_2, w_3\}\rangle.
\end{aligned}
$$

The unique signalling strategy that solves this game:

S	100%	$>50\%$	$\leq 50\%$
	$A(all)$	$A(most)$	$A(some)$

The Natural Information carried by utterance A(some)

- The branches allowed by strategy S:

  $\langle w_1, A(\text{all}), \{w_1\}\rangle$

  $\langle w_2, A(\text{most}), \{w_1, w_2\}\rangle$

  $\langle w_3, A(\text{some}), \{w_1, w_2, w_3\}\rangle$

- Natural information carried by A(some):

  $\{\langle w_3, A(\text{some}), \{w_1, w_2, w_3\}\rangle\}$

Hence: An utterance of A(some) is a true sign that less than 50% came to the party.

Calculating Implicatures in Signalling Games

The General Framework

As Signalling Game

A signalling game is a tuple:

$$\langle N, \Theta, p, (A_1, A_2), (u_1, u_2) \rangle$$

- N: Set of two players S,H.
- Θ: Set of types representing the speakers private information.
- p: A probability measure over Θ representing the hearer's expectations about the speaker's type.

- (A_1,A_2): the speaker's and hearer's action sets:
  - A_1 is a set of forms $\mathcal{F}$ / meanings $\mathcal{M}$.
  - A_2 is a set of actions.
- (u_1,u_2): the speaker's and hearer's payoff functions with

$$u_i: A_1 \times A_2 \times \Theta \to \mathbf{R}$$

Strategies in a Signalling Game

- Let $[\ ] : \mathcal{F} \rightarrow \mathcal{M}$ be a given semantics.
- The speaker's strategies are of the form:

$S : \Theta \rightarrow A_1$ such that

$$\boxed{S(\theta) = F \Rightarrow \theta \subseteq [F]}$$

i.e. if the speaker says F, then he knows that F is true.

Definition of Implicature

Given a signalling game as before, then an implicature

$$F +> \psi$$

is explained iff the following set is a subset of $[\psi] = \{w \in \Omega |\ w \models \psi\}$:

$$\{w \in \Omega | \exists \text{ solution } (S, H) \exists \theta \in \Theta : w \in \theta \wedge S(\theta) = F\}$$

Application

1. In the following we apply this criterion to calculating implicatures of answers.
2. The definition depends on the method of finding solutions.

- We present a method for calculating optimal answers.
- The resulting signalling and interpretation strategies are then the solutions we use for calculating implicatures.

Optimal Answers

Core Examples

Italian Newspaper

Somewhere in the streets of Amsterdam...

a) J: Where can I buy an Italian newspaper?
b) E: At the station and at the Palace but nowhere else. (SE)
c) E: At the station. (A) / At the Palace. (B)

- The answer (SE) is called **strongly exhaustive**.
- The answers (A) and (B) are called **mention–some answers**.

➢ A and B are as good as SE or as $A \wedge \neg B$:

d) E: There are Italian newspapers at the station but none at the Palace.

Partial Answers

If E knows only that ¬A, then ¬A is an optimal answer:

e) E: There are no Italian newspapers at the station.

If E only knows that the Palace sells foreign newspapers, then this is an optimal answer:

f) E: The Palace has foreign newspapers.

Partial answers may also arise in situations where speaker E has full knowledge:

- I: I need patrol for my car. Where can I get it?

  E: There is a garage round the corner.

- J: Where can I buy an Italian newspaper?

  E: There is a news shop round the corner.

The Framework

Support Problem

Definition: A support problem is a five–tuple $(\Omega, P_E, P_I, \mathcal{A}, u)$ such that

1. (Ω, P_E) and (Ω, P_I) are finite probability spaces,
2. $(\Omega, P_I, \mathcal{A}, u)$ is a decision problem.

We call a support problem well–behaved if

- for all $A \subseteq \Omega$: $P_I(A) = 1 \Rightarrow P_E(A) = 1$ and

Support Problem

$$
\begin{tikzcd}
\text{Expert E answers} \arrow[d] & \text{I decides for action} \arrow[d] & \text{Evaluation} \arrow[d] \\
\bullet \arrow[r, "A"] & \bullet \arrow[r, "a"] & \bullet \\
\begin{array}{c}\text{expectations} \\ \text{of E} \\ (\Omega, P_E)\end{array} \arrow[u] & \begin{array}{c}\text{expectations} \\ \text{of I} \\ (\Omega, P_I)\end{array} \arrow[u] & \begin{array}{c}\text{utility} \\ \text{function} \\ u(v, a)\end{array} \arrow[u]
\end{tikzcd}
$$

I's Decision Situation

I optimises expected utilities of actions:

$$EU(a) = \sum_{v \in \Omega} P(v) \times u(v, a).$$

After learning A, *I* has to optimise:

$$EU(a, A) = \sum_{v \in \Omega} P(v|A) \times u(v, a).$$

- I will choose an action a_A that optimises expected utility, i.e. for all actions b

$$EU(b,A) \leq EU(a_A,A)$$

  Given answer A, $H(A) = a_A$.
- For simplicity we assume that I's choice a_A is commonly known.

E’s Decision Situation

E optimises expected utilities of answers:

$$EU_E(A) := \sum_{v \in \Omega} P_E(v) \times u(v, a_A).$$

- (Quality): The speaker can only say what he thinks to be true.
- (Quality) restricts answers to:

$$Adm_S := \{A \subseteq \Omega \mid P_E(A) = 1\}$$

- Hence, E will choose his answers from:

$$\mathrm{Op}_{\mathcal{S}} = \{A \in Adm_S | \forall B \in Adm_S \, EU_E(B) \leq EU_E(A)\}.$$

Examples

The Italian Newspaper Examples

Italian Newspaper

Somewhere in the streets of Amsterdam...

a) J: Where can I buy an Italian newspaper?
b) E: At the station and at the Palace but nowhere else. (SE)
c) E: At the station. (A) / At the Palace. (B)

Possible Worlds (equally probable)

	Station	Palace
w_1	+	+
w_2	+	-
w_3	-	+
w_4	-	-

Actions and Answers

- I's actions:
  - a: going to station;
  - b: going to Palace;
- Answers:
  - A: at the station ($A = \{w_1, w_2\}$)
  - B: at the Palace ($B = \{w_1, w_3\}$)

- Let utilities be such that they only distinguish between success (value 1) and failure (value 0).
- Let's consider answer $A = \{w_1, w_2\}$.
- Assume that the speaker knows that A, i.e. there are Italian newspapers at the station.

The Calculation

- If hearing *A* induces hearer to choose *a* (i.e. $a_A=a$ 'going to station'):

$$EU_E(A) = \sum_{v \in \Omega} P_E(v) \times u(v, a_A) = \sum_{v \in A} P_E(v) \times u(v, a) = 1$$

- If hearing *A* induces hearer to choose *b* (i.e. $a_A=b$ 'going to Palace'):
  - If $P_E(B) = 1$, then $EU_E(A) = EU_E(b) = 1$.
  - $P_E(B) < 1$ leads to a contradiction.

$P_E(B) < 1$ leads to a contradiction:

1. $a_A = b$ implies $EU_I(b|A) \geq EU_I(a|A) = 1$.
2. Hence, $EU_I(b|A) = \sum_{v \in A} P_I(v)\, u(v,b) = 1$.
3. Therefore $P_I(B|A) = 1$, hence $P_I(B \cap A) = P_I(A)$, hence $P_I(A \backslash B) = 0$.
4. $P_E(A \backslash B) = 0$, due to well-behavedness.
5. $P_E(B \cap A) = P_E(A) = 1$, hence $P_E(B) = 1$.

Case: Speaker knows that Italian newspaper are at both places

- Calculation showed that $EU_E(A) = 1$.
- Expected utility cannot be higher than 1 (due to assumptions).
- Similar: $EU_E(B) = 1$; $EU_E(A \wedge B) = 1$.
- Hence, all these answers are equally optimal.

More Cases

- E knows that A and B:

  $EU_E(A) = EU_E(B) = EU_E(A \wedge B)$
- E knows that A and $\neg B$:

  $EU_E(A) = EU_E(A \wedge \neg B)$
- E knows only that A:

  For all admissible C: $EU_E(C) \leq EU_E(A)$

Implicatures of Answers

Signalling game associated to support problem (not unique!)

- (Ω, P_E, P_I, A, u): given support problem.
- $\langle N, \Theta, p, (A_E, A_I), (u_E, u_I) \rangle$: signalling game (to be defined).
- Assumption: $\exists$ K $P_E(X) = P_I(X|K)$.
  - $\Theta := \{K \subseteq \Omega \mid \forall\, v \in K\; P_I(v) > 0\}$
  - $A_I := A$
  - $u_I(A,a,K) := EU_I(a_A|K)$
  - $u_E(A,a,K) := EU_E(a_A|K)$
  - p arbitrary.

Definition of Implicature

Given a signalling game an implicature

$$F \mathrel{+\!>} \psi$$

is explained iff the following set is a subset of $[\psi] = \{w \in \Omega \mid w \models \psi\}$:

$$\{w \in \Omega \mid \exists \text{ solution } (S, H) \exists \theta \in \Theta : w \in \theta \wedge S(\theta) = F\}$$

The Criterion

- (Ω, P_E, P_I, A, u): given support problem.

Let

- $O(a) = \{w \in \Omega \mid \forall b \in \mathcal{A}\ u(a,w) \geq u(b,w)\}$.
- $\Omega^* := \{w \in \Omega \mid P_I(w) > 0\}$

➢ If it is common knowledge that

$$\boxed{\exists a \in \mathcal{A}\ P_E(O(a)) = 1}$$

then

$$\boxed{\Omega^* \cap O(a_A) \subseteq [\![\psi]\!] \Rightarrow A +\!> \psi.}$$

Glossary

$$O(a) = \{w \in \Omega \mid \forall b \in \mathcal{A}\; u(a, w) \geq u(b, w)\}.$$

- Set of worlds where *a* is optimal.

$$\Omega^* := \{w \in \Omega \mid P_I(w) > 0\}$$

- ‘Common Ground’

$$\exists a \in \mathcal{A}\; P_E(O(a)) = 1$$

- The expert knows an optimal action.

Examples

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The Italian Newspaper Examples

It holds:

- non A +> $\neg$ B

  $O(a_A) = \{w_1, w_2\}$, hence $O(a_A) \not\subset$ B* $= \{w_2, w_4\}$.

- non B +> $\neg$ A

  $O(a_B) = \{w_2, w_3\}$, hence $O(a_B) \not\subset$ A* $= \{w_3, w_4\}$.

Hip Hop at Roter Salon

John loves to dance to Salsa music and he loves to dance to Hip Hop but he can't stand it if a club mixes both styles.

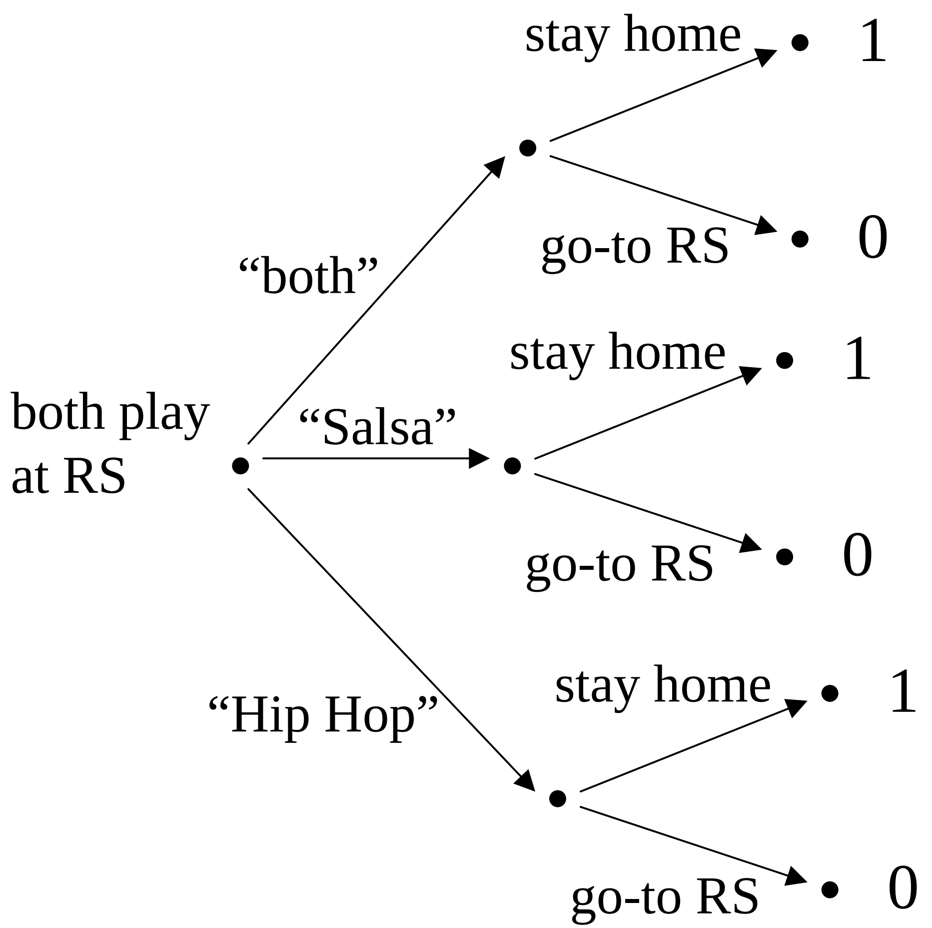
J: I want to dance tonight. Is the Music in Roter Salon ok?

E: Tonight they play Hip Hop at the Roter Salon.

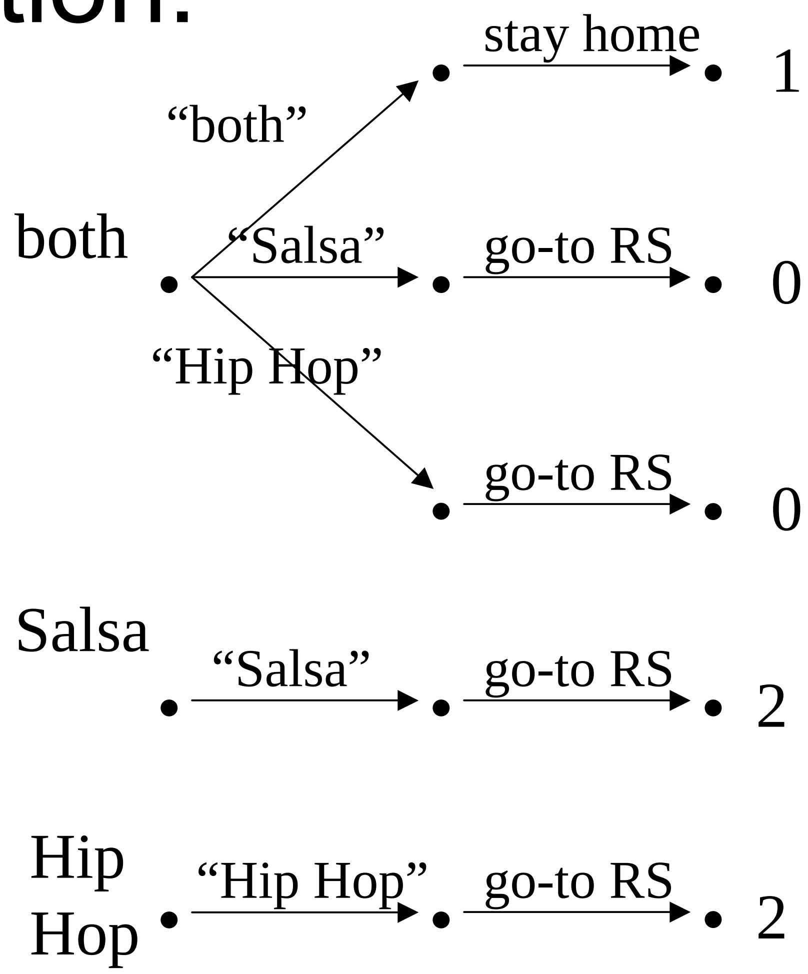
+> They play only Hip Hop.

A game tree for the situation where both Salsa and Hip Hop are playing

RS = Roter Salon



After the first step of backward induction:



After the second step of backward induction:

both $\bullet \xrightarrow{\text{“both”}} \bullet \xrightarrow{\text{stay home}} \bullet$ 1

Salsa $\bullet \xrightarrow{\text{“Salsa”}} \bullet \xrightarrow{\text{go-to RS}} \bullet$ 2

Hip Hop $\bullet \xrightarrow{\text{“Hip Hop”}} \bullet \xrightarrow{\text{go-to RS}} \bullet$ 2

In all branches that contain “Salsa” the initial situation is such that only Salsa is playing at the Roter Salon.

Hence: “Salsa” **implicates** that only Salsa is playing at Roter Salon

Hip Hop at Roter Salon

Abbreviations:

$H(x)$: There is Hip Hop at x;

$S(x)$: There is Salsa at x.

- Good(x) := $(H(x) \vee S(x)) \wedge \neg(H(x) \wedge S(x))$

Assumptions

I. Equal Probabilities

$$\exists h > 0 \, \forall x \, P_I(H(x)) = h;$$

$$\exists s > 0 \, \forall x \, P_I(S(x)) = s;$$

$$\exists g > 0 \, \forall x \, P_I(Good(x)) = g.$$

II. Independence: $X, Y \in \{H, S, Good\}$

$$a \neq b \Rightarrow P_I(X(a) \wedge Y(b)) = P_I(X(a)) \times P_I(Y(b)).$$

III. Learning $H(x)$ or $S(x)$ raises expected utility of going to salon x:

a) EU_I(going-to-x) < EU_I(stay-home) < EU_I(going-to-$x|H(x)$)

b) EU_I(going-to-x) < EU_I(stay-home) < EU_I(going-to-$x|S(x)$)

Violating Assumptions II

The Roter Salon and the Grüner Salon share two DJs. One of them only plays Salsa, the other one mainly plays Hip Hop but mixes into it some Salsa. There are only these two Djs, and if one of them is at the Roter Salon, then the other one is at the Grüner Salon. John loves to dance to Salsa music and he loves to dance to Hip Hop but he can’t stand it if a club mixes both styles.

J: I want to dance tonight. Is the Music in Roter Salon ok?

E: Tonight they play Hip Hop at the Roter Salon.