Signalling Games and Pragmatics
Day V

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The Course

- Day I: Introduction: From Grice to Lewis
- Day II: Basics of Game and Decision Theory
- Day III: Two Theories of Implicatures (Parikh, Jäger)
- Day IV: Best Answer Approach
- **Day V: Utility and Relevance**

Game and Decision Theoretic Approaches to Gricean Pragmatics

Distinguish between Approaches based on:

- Classical Game Theory
  - Underspecification based Approach (P. Parikh).
  - Information Based Approach (Benz).
- Evolutionary Game Theory
  - E.g. v. Rooij, Jäger
- **Decision Theory**
  - **Relevance base approaches**
  - **v. Rooij**

Utility and Relevance

Day V – August, 11th

Overview

- Relevance and Implicatures
- Relevance in Decision Theory
- Relevance and Best Answers
- Implicatures and Relevance Scales
- Calculating Implicatures and Relevance

Three Negative Results

- No relevance based approach can avoid non- optimal answers.
- There are ‘*relevance implicatures*’ of answers that cannot be accounted for by any approach based on relevance scales.
- The appropriate relevance principle for calculating ‘*relevance implicatures*’ is not a conversational maxim.

Relevance and Implicatures

Maxim of Relevance:

Make your contributions relevant.

Maxim of Quantity:

1. Make your contribution to the conversation as informative as is required for he current talk exchange.
2. Do not make your contribution to the conversation more informative than necessary.

The Conversational Maxims
(without Manner)

- Be truthful (Quality) and say as much as you can (Quantity) as long as it is relevant (Relevance).

Scalar Implicatures
(Quantity Implicature)

- Let A(x) be a sentence frame.
- $\langle e_1, e_2, \ldots, e_n \rangle$ is a scale iff
  - $e_1, e_2, \ldots, e_n$ are elements of a closed lexical category.
  - for i<j: $A(e_i) \Rightarrow A(e_j)$ but $\neg A(e_j) \Rightarrow A(e_i)$.
- then for i<j: $A(e_j)$ +> $A(e_i)$
- Example: ⟨all, most, many, some⟩

Relevance Scale Approach

- Let $\mathcal{M}$ be a set of propositions.
- Let $\leq$ be a linear well-founded pre-order on $\mathcal{M}$ with interpretation:

  $A \leq B \Leftrightarrow$ B is at least as relevant as A.

- then A +> B iff $A < B$.

The Italian Newspaper Example

Somewhere in the streets of Amsterdam...

a) J: Where can I buy an Italian newspaper?
b) E: At the station and at the Palace but nowhere else. (SE)
c) E: At the station. (A) / At the Palace. (B)

The Out of Patrol Example

A stands in front of his obviously immobilised car.

A: I am out of petrol.

B: There is a garage around the corner. (G)

+> The garage is open (H)

A “standard” explanation

Set H*:= The negation of H

- B said that G but not that H*.
- H* is relevant and $G \wedge H^* \Rightarrow G$.
- Hence if $G \wedge H^*$, then B should have said $G \wedge H^*$ (Quantity).
- Hence H* cannot be true, and therefore H.

Problem: We can exchange H and H* and still get a *valid* inference:

1. B said that G but not that H.
2. H is relevant and $G \wedge H \Rightarrow G$.
3. Hence if $G \wedge H$, then B should have said $G \wedge H$ (Quantity).
4. Hence H cannot be true, and therefore H*.

Relevance and Answers
(widely accepted statements)

Relevance

- defines a linear pre-order on a set of possible answers.
- is presumed to be maximised by the answering person.
- makes the ‘standard’ explanation in the out-of-patrol example valid.
- is defined from the receivers perspective.

Relevance in Decision Theory

Game and Decision Theory

- **Decision theory:** Concerned with decisions of **individual** agents
- **Game theory:** Concerned with interdependent decisions of **several** agents.

Measures of Relevance I

New information A is relevant if

- it leads to a different choice of action, and
- it is the more relevant the more it increases thereby expected utility.

Measures of Relevance I

- Let ((Ω, P),A,u) be a given decision problem.
- Let a* be the action with maximal expected utility before learning *A*.

Possible definition of **Relevance** of *A*:

$$UV(A) = \max_{a \in \mathcal{A}} EU(a|A) - EU(a^*|A).$$

(Sample Value of Information)

Measures of Relevance II

New information A is relevant if

- it increases expected utility.
- it is the more relevant the more it increases it.

$$UV'(A) = \max_{a \in \mathcal{A}} EU(a|A) - \max_{a \in \mathcal{A}} EU(a).$$

Measures of Relevance III

New information A is relevant if

- it changes expected utility.
- it is the more relevant the more it changes it.

$$UV''(A) = \left| \max_{a \in \mathcal{A}} EU(a|A) - \max_{a \in \mathcal{A}} EU(a) \right|.$$

A Decision Problem

- An oil company has to decide where to build a new oil production platform.
  - Given the current information it would invest the money and build the platform at a place off the shores of Alaska.
  - An alternative would be to build it off the coast of Brazil.
- Build a platform off the shores of Alaska. (act a)
- Build it off the shores of Brazil. (act b)

- The company decides for exploration drilling.
- Using sample value of information means:
  - Only if the exploration drilling gives hope that there is a larger oil field off the shores of Brazil, the company got relevant information.

Relevant information $\neq$ desired information

- Using utility value *UV'* means:
  - Only if the exploration drilling rises the expectations about the amount of oil, the company got relevant information.
- Using utility value *UV''* means:
  - The more the exploration drilling changes expectations about the amount of oil the more relevant is the result to the company.

Relevance and Best Answers

General Situation

We consider situations where:

- A person *I*, called inquirer, has to solve a decision problem $((\Omega, P), A, u)$.
- A person *E*, called expert, provides *I* with information that helps to solve *I*'s decision problem.
- P_E represents E's expectations about Ω at the time when she answers.

Assumption: E optimises the relevance of her answers.

Who’s probability is P?

Three possibilities:

1. It is the inquirer’s subjective probability.
2. It is the expert’s subjective probability.
3. It is the subjective probability that *E* assigns to *I*.

=> Only 3. is reasonable.

An Example

Assume that it is common knowledge between I and E that there are Italian newspapers at the station with probability 2/3, and at the Palace with probability 1/3.

- Now, E learned privately that they are in stock at both places.

What should E answer to:

Where can I buy an Italian newspaper?

Answers

(A) There are Italian newspapers at the station.

(B) There are Italian newspapers at the Palace.

- With sample value of information: Only B is relevant.
- With utility value: A, B, and A$\wedge$B are equally relevant.

- Assume now that E learned that:
  - (¬A) there are no Italian newspapers at the station.
- With sample value of information: ¬A is relevant.
- With utility value: the uninformative answer is the most relevant answer.

Need: Uniform definition of relevance that explains all examples.

Partial Answers

We consider only sample value of information as measure of relevance.

Examples with non-trivial partial answers

There is a strike in Amsterdam and therefore the supply with foreign newspapers is a problem. The probability that there are Italian newspapers at the station is slightly higher than the probability that there are Italian newspapers at the Palace, and it might be that there are no Italian newspapers at all. All this is common knowledge between I and E.

- Now E learns that

  (N) the Palace has been supplied with foreign newspapers.

- In general, it is known that the probability that Italian newspapers are available at a shop increases significantly if the shop has been supplied with foreign newspapers.

We describe the epistemic states by:

$P_I(A) > P_I(B)$ and $P_X(B \cap N) > P_X(A \cap N)$ for $x = I, E$.

It follows that going to the Palace (*b*) is preferred over going to the station (*a*):

$$EU_I(a, N) = \sum_{v \in N} P_I(v|N) \times u(v, a) = P_I(A \cap N);$$

$$EU_I(b, N) = \sum_{v \in N} P_I(v|N) \times u(v, b) = P_I(B \cap N).$$

- **Sample Value of Information: *N* is relevant.**

First Modification

- We assume the same scenario as in before but E learns this time that

(M) the Palace has been supplied with British newspapers.

Due to the fact that the British delivery service is rarely affected by strikes and not related to newspaper delivery services of other countries, this provides no evidence whether or not the Palace has been supplied with Italian newspapers.

- M provides no evidence whether or not there are Italian newspaper at the station (A) or the Palace (B)
- We assume therefore:

$$P_E(A) = P_E(M \cap A) > P_E(M \cap B) = P_E(B).$$

- $M \subseteq N$: Hence E knows N. Is N still a good answer?
- *I*'s epistemic state hasn't changed

Sample Value of Information: *N* is still relevant.

Second Modification

We assume the same scenario as before where *E* learns that

(N) the Palace got supplied with foreign newspapers

but

- **her intuition tells her that, if there have been Italian newspapers among them, then they are sold out before *I* can get there.**

Of course, this is only a conjecture of hers.

- Again, *I*'s epistemic state hasn't changed:

  Sample Value of Information: *N* **is still relevant.**

- Hence, Grice' relevance maxim defined by sample value of information leads in these cases to a **misleading answer**!

No relevance based approach can avoid non-optimal answers.

First Negative Result about Relevance

Some Definitions

- A **support problem** is a five–tuple (Ω, P_E, P_I, A, u) such that (Ω, P_E) and (Ω, P_I) are finite probability spaces and $((\Omega, P_I), A, u)$ is a decision problem.
- A support problem **well–behaved** if:

  For all $A \in \Omega$: $P_I(A) = 1 \Rightarrow P_E(A) = 1$

- For a given support problem S = (Ω, P_E, P_I, A, u) let $D_S := ((\Omega, P_I), A, u)$ and

$$Adm_S := \{A \subseteq \Omega \mid P_E(A)=1\}$$

- Let $\mathcal{S}$ denote the set of all support problems.
- We set: $\mathcal{D} := \{(D_S, Adm_S) \mid S \in \mathcal{S}\}$

General Assumptions

- A support problem S represents an answering situation; implies (Coop).
- (Quality) The answering expert can only choose answers from Adm_S.
- (Utility) Interlocutors are utility maximisers.
- For each support problem, the inquirer's choice of action is predictable; i.e. for each S there is a commonly known function that describes his choice of action when receiving information A:

$$a : Adm_S \longrightarrow \mathcal{A},\ A \mapsto a_A.$$

Relevance Based Decision Functions

We call a function

$$R: \mathcal{D} \to \wp(\Omega)$$

a **relevance based** (non-argumentative) **decision function**.

- Point: Every decision theoretic explication of Grice maxim of relevance will define a relevance based decision function that tells the answering expert which answer she should give.

- **Misleading Answer:**

  Let $\langle \Omega, P_E, P_I, \mathcal{A}, u \rangle$ *be a given support problem and* $a : Adm_S \longrightarrow \mathcal{A}$, $A \mapsto a_A$, *as above, then an answer* $A \subseteq \Omega$ *is* misleading, *iff* $EU_E(a_A) \neq \max_{a \in \mathcal{A}} EU_E(a)$.

- **Non-trivial partial answer:**

Let $S = \langle \Omega, P_E, P_I, \mathcal{A}, u \rangle$ be a given support problem, then S has a non–trivial partial answer *C, iff there exist actions $a, b \in \mathcal{A}$ and $a : \ Adm_S \longrightarrow \mathcal{A},\ A \mapsto a_A$, as above, such that for*

$$A := \{v \in \Omega \mid u(v, a) > u(v, b)\}$$
$$B := \{v \in \Omega \mid u(v, b) > u(v, a)\}$$

it holds that (1) $P_X(C), P_X(A|C), P_X(B|C) > 0$, for $x = I, E$, and (2) $a = a_C$.

- **Theorem**

Let $S = \langle \Omega, P_E, P_I, \mathcal{A}, u \rangle$ *be a well–behaved support problem with a non–trivial partial admissible answer* C. *Then there exists a probability distribution* P'_E *on* Ω *such that for the support problem* $S' = \langle \Omega, P'_E, P_I, \mathcal{A}, u \rangle$

1. *the admissible answers are the same as for* S,
2. C *is a misleading admissible answer.*

- **Corollary:**

Let $\mathcal{S}$ be the set of all support problems over Ω. For $S \in \mathcal{S}$ let D_S denote its associated decision problem. Let $\mathcal{D} := \{\langle D_S, Adm_S \rangle \mid S \in \mathcal{S}\}$ where Adm_S is the set of admissible answers of S. Then there exists no function $R : \mathcal{D} \longrightarrow \mathcal{P}(\Omega)$ such that for all $S \in \mathcal{S}$: $R(D_S, Adm_S) \in \mathrm{Op}_S$.

Implicatures and Relevance Scales

Second Negative Result about Relevance

- A theory about relevance implicatures is a **relevance scale approach** iff it defines or postulates a linear pre-order $\prec$ on propositions such that an utterance of proposition *A* implicates a proposition *H* iff *A* is less relevant than $\neg$ *H*:

$$A \prec \neg H \Leftrightarrow A +> H$$

Lemma

No relevance scale approach can satisfy the following set of implicatures:

1. $A_1 +> H_1$;
2. $A_2 +> H_2$;
3. not $A_1 +> \neg A_2$;
4. not $A_2 +> \neg A_1$;
5. not $A_1 +> H_2$.

Where it is assumed that A_1, A_2, H_1, and H_2 are pair-wise distinct propositions.

Proof:

1. $A_1 +\!> H_1$ implies $A_1 \prec \neg H_1$;
2. not $A_1 +\!> \neg A_2$ implies $A_1 \nprec A_2$;
3. not $A_2 +\!> \neg A_1$ implies $A_2 \nprec A_1$;
4. hence, $A_1 \approx A_2$ from the last two lines;
5. $A_2 +\!> H_2$ implies $A_2 \prec \neg H_2$;
6. hence, $A_1 \prec \neg H_2$ from lines 4 and 5;
7. not $A_1 +\!> H_2$ implies $A_1 \nprec \neg H_2$, in contradiction to line 6.

An Example
(Argentine wine)

- Somewhere in Berlin... Suppose J approaches the information desk at the entrance of a shopping centre.
- He wants to buy Argentine wine. He knows that staff at the information desk is very well trained and know exactly where you can buy which product in the centre.
- E, who serves at the information desk today, knows that there are two supermarkets selling Argentine wine, a Kaiser’s supermarket in the basement and an Edeka supermarket on the first floor.
- J: I want to buy some Argentine wine. Where can I get it?
- E: Hm, Argentine wine. Yes, there is a Kaiser’s supermarket downstairs in the basement at the other end of the centre.

Propositions

1. A_1: There is a Kaiser's supermarket in the shopping centre.
2. A_2: There is an Edeka supermarket in the shopping centre.
3. H_1: The Kaiser's supermarket sells Argentine wine.
4. H_1: The Edeka supermarket sells Argentine wine.

No Relevance scale approach can explain this example.

Calculating Implicatures and Relevance

Third Negative Result about Relevance

The Out of Patrol Example

A stands in front of his obviously immobilised car.

A: I am out of petrol.

B: There is a garage around the corner. (G)

+> The garage is open (H)

A “standard” explanation

Set H*:= The negation of H

- B said that G but not that H*.
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4. Hence H cannot be true, and therefore H*.

- Is there a relevance measure that makes the first but not the second argument valid?

The previous result shows that this is not possible if the relevance measure defines a linear pre-order on propositions.

The Posterior Sample Value of Information

If

1. the speaker said that A;
2. it is common knowledge that $\exists a\ P_E(O(a)) = 1$
3. for all $K \subseteq H^*$: $UV_I(K|A) > 0$,

then H is true.

Where $UV_I(a_K|A)$ is the sample value of information **posterior** to learning A.

$$UV_I(K|A) := EU_I(a_{A \cap K}|A \cap K) - EU_I(a_A|A \cap K)$$

Application to Out-of-Patrol Example

- Let $K \subseteq H^*$ = ‘the garage is closed’
- A: ‘there is a garage round the corner’
- We assume that the inquirer has a better alternative than going to a closed garage.
- It follows then that $UV_I(K|A) > 0$, and our criterion predicts that

  H: ‘the garage is open’

  is true.

Relevance and Answers

Relevance

- is presumed to be maximised by the answering person.
- defines a linear pre-order on the set of possible answers.
- is defined from the receivers perspective.
- makes the ‘standard’ explanation in the out-of-patrol example valid.

Relevance and Answers

Relevance

- **is presumed to be maximised by the answering person.**
- **defines a linear pre-order on a set of possible answers.**
- **is defined from the receivers perspective.**
- **makes the ‘standard’ explanation in the out-of-patrol example valid.**

Relevance and Conversational Maxim

Conversational Maxim:

- presumed to be followed by the speaker.
- Necessary for calculating appropriate answers and implicatures.

=> The relevance measure defined by the posterior sample value of information does not define a conversational maxim.

Summing Up

Game and Decision Theoretic Approaches to Gricean Pragmatics

Distinguish between Approaches based on:

- Classical Game Theory
  - Underspecification based Approach (P. Parikh).
  - Information Based Approach (Benz).
- Evolutionary Game Theory
  - E.g. v. Rooij, Jäger
- Decision Theory
  - Relevance base approaches
  - E.g. Merin, v. Rooij

THE END