Game Theoretic Pragmatics

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The Course

1. addresses topics of Gricean Pragmatics.
2. concentrates mainly on two frameworks:
   i. Iterated Best Response
   ii. Optimal Answer Model
3. is based on classical game theory.

4. not concerned with the evolution of language structure and use.

$\Rightarrow$ no evolutionary game theory!

Models of Signalling Behaviour

GT Models

- non–evolutionary
  - online
    - immediate effects
    - two interlocutors
  - competence
    - several round of interaction
    - two interlocutors
- evolutionary
  - replication involving several generations
  - large populations (infinite)

Models of Signalling Behaviour

GT Models

non–evolutionary

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Outline

1. Introduction and Motivation
2. The Basic Iterated Best Response Model
3. The Basic Optimal Answer Model
4. Aspects of Bounded Rationality
5. Some Extensions of the Optimal Answer Model

Game Theoretic Pragmatics

Day 1
Introduction and Motivation

Anton Benz

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16 August 2010

Outline

Section 1

Gricean Pragmatics and Game Theory

A simple picture of communication

1. The speaker encodes some proposition p.
2. He sends it to an addressee.
3. The addressee decodes it again and writes p in his knowledgebase.

Problem

$\Rightarrow$ Problem: We often communicate much more than we literally say!

Some students failed the exam.
$+>$ Most of the students passed the exam.

Communicated Meaning

Herbert Paul Grice (1913–1988), see [Grice, 1989]

Grice distinguishes between:

- What is said.
- What is implicated.

Example 1

"Some of the boys came to the party."

- said: at least two came
- implicated: not all came

Assumptions about Conversation

- Conversation is a cooperative effort;
- Each participant recognises a common purpose in the talk exchange.

Example 2

A stands in front of his obviously immobilised car.
A: I am out of petrol.
B: There is a garage round the corner.

$\Rightarrow$ Joint purpose of B's response: Solve A's problem of finding petrol for his car.

Implicatures

[Grice, 1989, p. 86]

What is an implicature?

"... what is implicated is what is required that one assume a speaker to think in order to preserve the assumption that he is observing the Cooperative Principle (and perhaps some conversational maxims as well), ..."

The Cooperative Principle

The Cooperative Principle

“Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged.” [Grice, 1989]

The Conversational Maxims

1. *The Maxim of Quality*: Try to make your contribution one that is true, specifically:
    1. Do not say that you believe to be false.
    2. Do not say that for which you lack adequate evidence.
2. *The Maxim of Quantity*:
    1. Make your contribution as informative as is required by the current purpose of the exchange.
    2. Do not make your contribution more informative than is required.
3. *The Maxim of Relation*: Make your contribution relevant.
4. *The Maxim of Manner*: Be perspicuous, and specifically:
    1. avoid obscurity,
    2. avoid ambiguity,
    3. be brief,
    4. be orderly.

Examples of Implicatures

(Quantity:)

1. John has five children.
   $+>$ *John has not more than five children.*
2. The flag is white.
   $+>$ *The flag is white all over.*

(Relevance:)

3. A: Smith doesn't seem to have a girlfriend these days.
   B: He has been paying a lot of visits to New York lately.
   $+>$ *Smith presumably has a girlfriend in New York.*
4. A is writing a testimonial about a pupil who is a candidate for a philosophy job. A writes: "Dear Sir, Mr. X's command of English is excellent, and his attendance at tutorials has been regular. Yours etc."
   $+>$ *Mr. X is no good in philosophy.*

Examples of Implicatures

(Manner:)

5. Open the door!
   Walk up to the door, turn the door handle clockwise as far as it will go, and then pull gently toward you.
   $+>$ *Pay special attention to what you are doing!*
6. Miss Singer sang an aria from Rigoletto.
   Miss Singer produced a series of sounds corresponding closely to the score of an aria from Rigoletto.
   $+>$ *Miss Singer's performance was very bad.*

The Conversational Maxims

short, without Manner

Maxim of Quality: Be truthful!

Maxim of Quantity:

- Say as much as you can.
- Say no more than you must.

Maxim of Relevance: Be relevant!

The Conversational Maxims

very short, without Manner

(QQR)

Be truthful (Quality) and say as much as you can (Quantity) as long as it is relevant (Relevance).

An Application

A Case of a Scalar Implicature

Example 3

"Some of the boys came to the party."

- said: at least two came
- implicated: not all came

An Explanation based on Maxims

Let A(x) $\equiv$ "*x* of the boys came to the party."

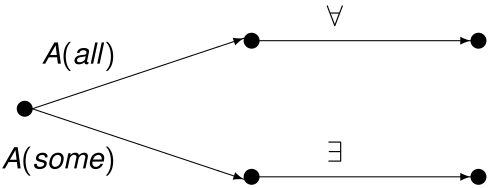
1. The speaker had the choice between the forms A(all) and A(some).
2. A(all) is more informative than A(some) and the additional information is also relevant.
3. Hence, if all of the boys came, then A(all) is preferred over A(some) (Quantity) + (Relevance).

4. The speaker said A(some).
5. Hence it cannot be the case that all came.
6. Therefore some but not all came to the party.

A Graphical Interpretation

Situation: All of the boys came to the party.

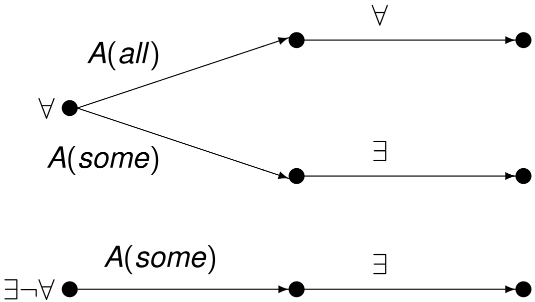
1. The speaker has a choice between A(all) and A(some).
2. If he chooses A(all), the hearer has to interpret *all* by the universal quantifier.
3. If he chooses A(some), the hearer has to interpret *some* by the existential quantifier.



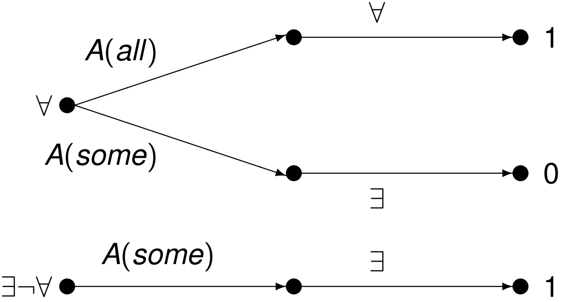
Adding Alternative Situation

Alternative Situation: Some but not all came.

4. If he chooses A(some), the hearer has to interpret *some* by the existential quantifier.



Adding Speaker's Preferences



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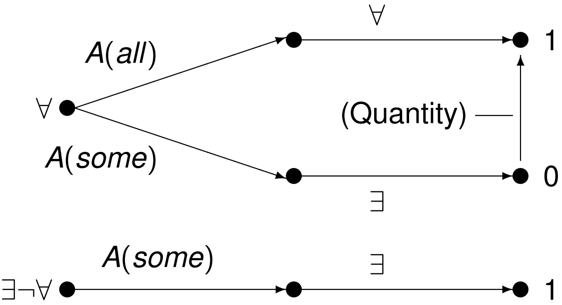


Simplifying the Tree

Eliminate all dominated speaker's choices

After elimination of all branches which the speaker will not choose:

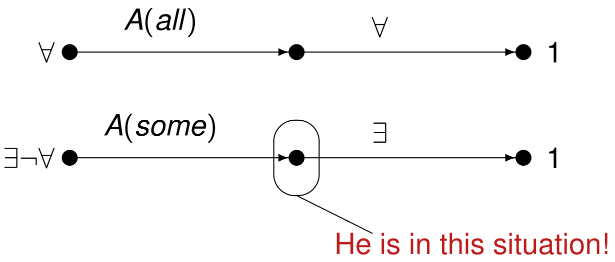
$$\forall \bullet \xrightarrow{A(all)} \bullet \xrightarrow{\forall} \bullet \ 1$$

$$\exists\neg\forall \bullet \xrightarrow{A(some)} \bullet \xrightarrow{\exists} \bullet \ 1$$

Simplifying the Tree

Eliminate all dominated speaker's choices

Hence, the hearer can infer from an utterance of A(some):



Section 2

Game and Decision Theory

Remark

The situation depicted in the graph for scalar implicatures the outcome depends on the decision of the speaker only!

- Decision theory: decisions of individual agents
- Game theory: interdependent decisions of several agents

$\Rightarrow$ Choice of optimal speaker strategy was a problem of decision theory!

Decision Theory

If a decision only depends on

- the state of the world,
- the actions to choose from and
- their outcomes

but not on

- the choice of actions by other agents,

then the problem belongs to decision theory.

Game Theory

A game is being played by a group of individuals whenever the fate of an individual in the group depends not only on his own actions but also on the actions of the rest of the group. [Binmore, 1990]

Game Theory and Pragmatics

- In a very general sense we can say that we play a game together with other people whenever we have to decide between several actions such that the decision depends on:
  - the choice of actions by others
  - our preferences over the ultimate results.
- Whether or not an utterance is successful depends on
  - how it is taken up by its addressee
  - the overall purpose of the current conversation.

Section 3

Why a New Framework

Why a New Framework?

- Basic concepts of Gricean pragmatics are undefined, most notably the concept of relevance.
- On a purely intuitive level, it is often not possible to decide whether an inference of an implicature is correct or not.

Out-of-Petrol Example

A stands in front of his obviously immobilised car:

A: I am out of petrol.

B: There is a garage round the corner. (*G*)

$+\!>$ The garage is open. (*R*)

A possible Explanation

Set $R^* :=$ The negation of R.

1. B said that G but not that R^*.
2. R^* is relevant and $G \wedge R^* \Rightarrow G$.
3. Hence, if $G \wedge R^*$, then B should have said $G \wedge R^*$ (Quantity).
4. Hence, R^* cannot be true, and therefore R.

Problem: A Second Valid Explanation

Exchange R and R^*

1. B said that G but not that R.
2. R is relevant and $G \wedge R \Rightarrow G$.
3. Hence, if $G \wedge R$, then B should have said $G \wedge R$ (Quantity).
4. Hence, R cannot be true, and therefore R^*.

Problem

- As in the second step both R and R^* can be called relevant, it cannot be decided which explanation is correct.
- All decision theoretic standard measures of relevance would predict that R is relevant, and hence that R^* should be implicated (which is wrong).

$\Rightarrow$ Without definition of relevance it is not possible to decides whether a typical explanation of a relevance implicature is in fact valid or not.

Section 4

A Graphical Solution to the Out–of–Petrol Example

Out-of-Petrol Example

Modified version

Example 4

A stands in front of his obviously immobilised car:

A: I am out of petrol.

B: There is a garage to the left round the corner. (G_l)

Possible alternative:

- There is a garage to the right round the corner. (G_r)

Properties:

- Garage can be **open** and **closed**.

First Steps Towards a Model

World	Garage left	Garage right	Action g_l	Action g_r	random search r
w_1	open	open	1	1	ε
w_2	open	closed	1	0	ε
w_3	open	—	1	0	ε
w_4	closed	open	0	1	ε
w_5	closed	closed	0	0	ε
w_6	closed	—	0	0	ε
w_7	—	open	0	1	ε
w_8	—	closed	0	0	ε
w_9	—	—	0	0	ε

Meaning:

- open: at this place there is a garage and it is open.
- closed: at this place there is a garage and it is closed.
- — : at this place there is no garage.

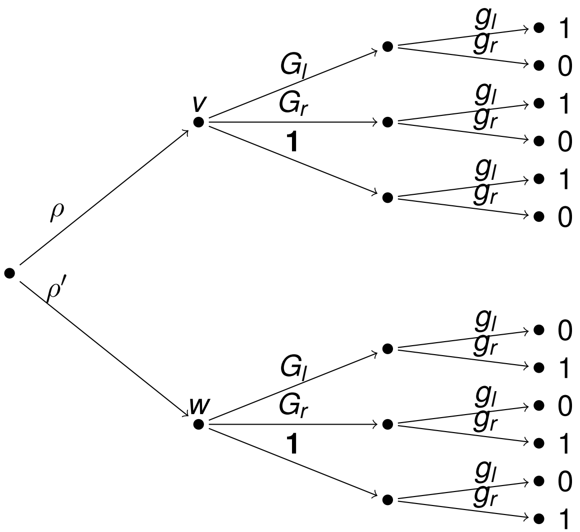
Some Simplifications for the Graphical Solution

- Next, we provide a graphical solution.
- We simplify the trees by considering the following worlds only:

World	Garage left	Garage right	Probab.	Action g_l	Action g_r	random search r
$v = w_2$	open	closed	ρ	1	0	ε
$w = w_4$	closed	open	ρ'	0	1	ε

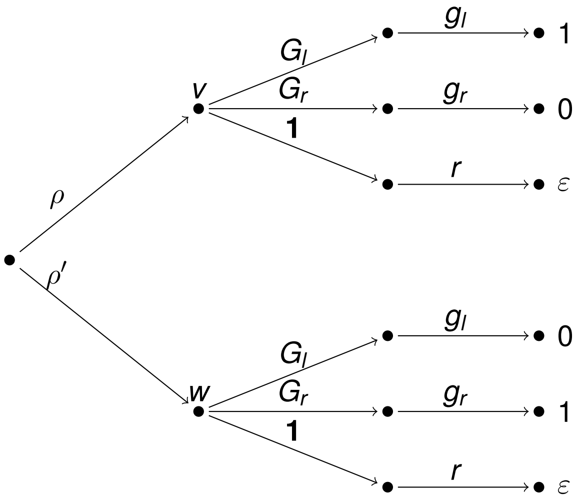
In the first tree, we simplify further by omitting random search r.

The Game Tree



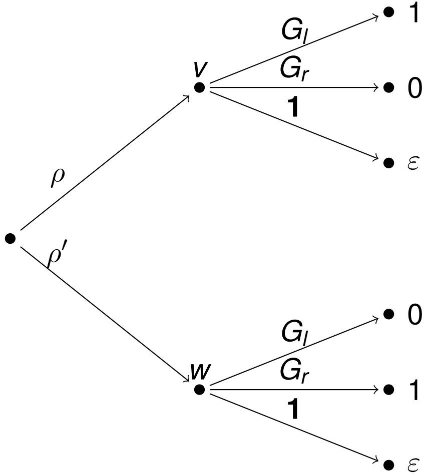
The Game Tree

Hearer chooses optimal act.



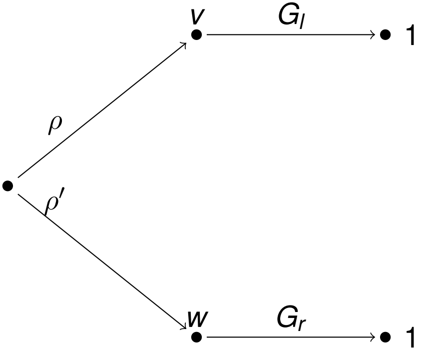
The Game Tree

Speaker calculating backwards



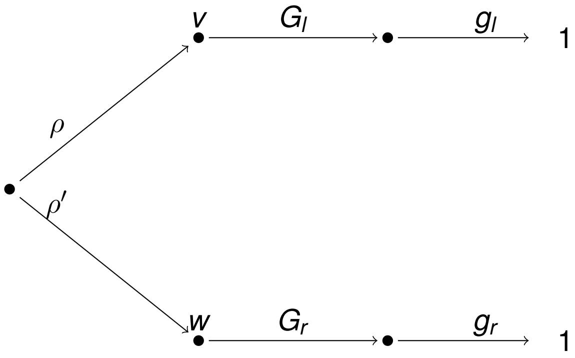
The Game Tree

Speaker choosing optimal action



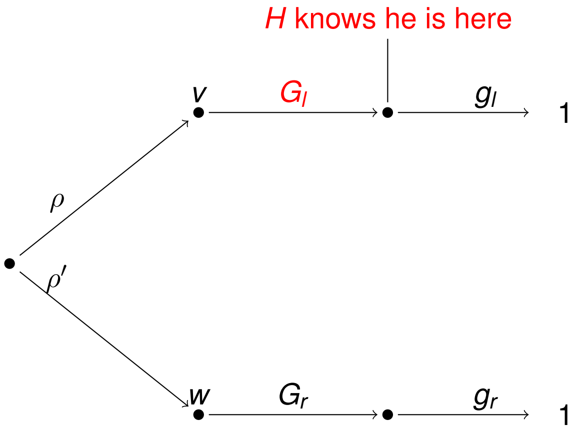
The Game Tree

Predicted behaviour



The Game Tree

Hearer can infer that the speaker is in v when uttering G_l!



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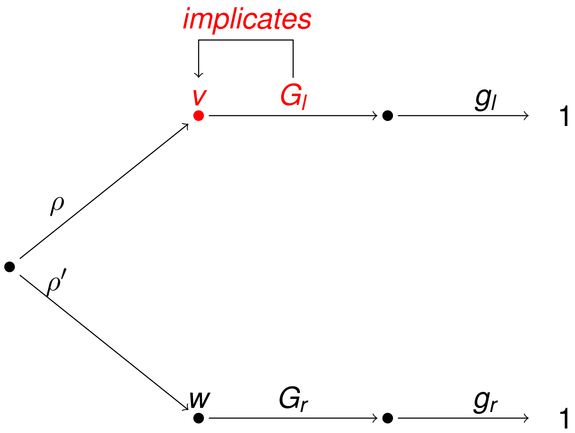


Assumptions

Actions, worlds, probabilities, and utilities:

World	Garage left	Garage right	Probability	Action g_l	Action g_r	random search r
v	open	closed	ρ	1	0	ε
w	closed	open	ρ'	0	1	ε

Implicit Assumptions:

- Before learning anything: H chooses random search r.
- After learning that there is garage to the left: H chooses g_l.
- After learning that there is garage to the right: H chooses g_r.

Collecting the Elements

In our graphical model, we represented:

- Worlds
- Actions
- Utilities
- Probabilities

In addition, we like to have representations of:

- Information states of interlocutors.
- Decision rules for choosing between actions.
- The speaker's and hearer's strategies.

$\Rightarrow$ Introduce Signalling Games!

Section 5

Introducing Signalling Games

[Benz et al., 2006]

Static v.s. Dynamic Games

Some basic distinctions in game theory

- **Static game:** In a static game every player performs only one action, and all actions are performed simultaneously.
- **Dynamic game:** In dynamic game there is at least one possibility to perform several actions in sequence.

Normal Form v.s. Extensive Form

Some basic distinctions in game theory

- **Normal form:** Representation in matrix form.
- **Extensive form:** Representation in tree form. It is more suitable for dynamic games.

- The most important games for us are signalling games.
- They will be represented in extensive form.

Playing a Signalling Game

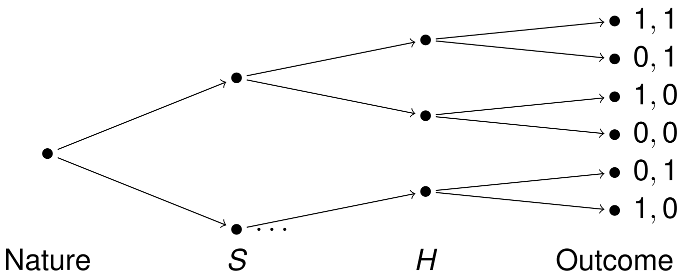
A signalling game is played in the following order:

1. Nature chooses a world with a certain probability.
2. An information state (his **type**) is assigned to each interlocutor.
3. The game starts with a message sent by the speaker.
4. After receiving the message, the hearer chooses an action from his action set.
5. This ends the game.

- An interlocutor's **type** represents his **private knowledge**.
- All other parameters of the game are assumed to be **common knowledge**.

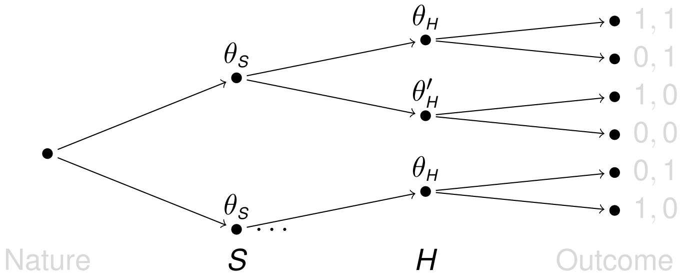
The Game Tree

1. The game tree shows three sequential moves: Nature, Speaker, Hearer, and their final payoffs.



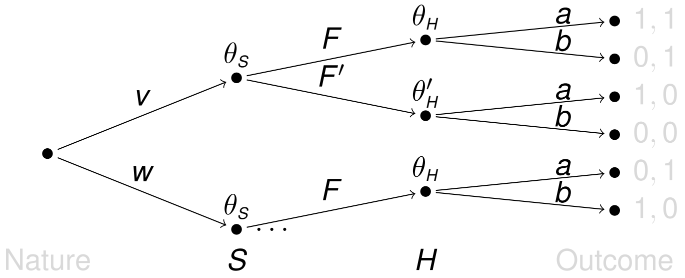
The Game Tree

1. The game tree shows three sequential moves: Nature, Speaker, Hearer, and their final payoffs.
2. The speaker's type θ_S and the hearer's type θ_H are assigned to the nodes at which they have to act.



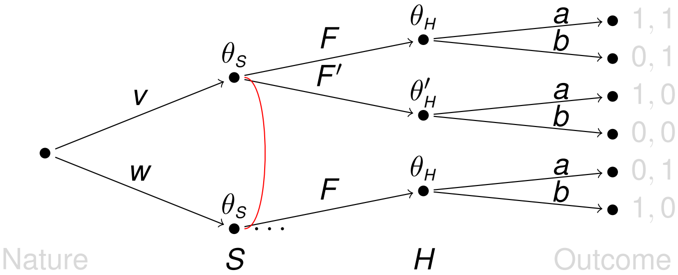
The Game Tree

1. The game tree shows three sequential moves: Nature, Speaker, Hearer, and their final payoffs.
2. The speaker's type θ_S and the hearer's type θ_H are assigned to the nodes at which they have to act.
3. The edges are labelled by the moves or acts of the players.



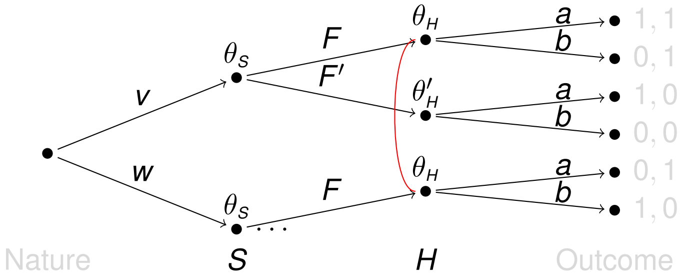
The Game Tree

1. Two nodes are indiscernible to the speaker if they are assigned the same type θ_S.



The Game Tree

1. Two nodes are indiscernible to the speaker if they are assigned the same type θ_S.
2. Two nodes are indiscernible to the hearer if they are assigned the same type θ_H and the same signal.



Probablilities

1. $P(v, \theta_S, \theta_H)$: Probability with which nature chooses world v, speaker type θ_S, and hearer type θ_H.
2. We can think of $P(v, \theta_S, \theta_H)$ as the result of first choosing v, and then simultaneously θ_S and θ_H:

$$P(v, \theta_S, \theta_H) = P(v) \times P(\theta_S, \theta_H | v) \quad (5.1)$$

$\Rightarrow$ Collecting all these elements in a structure leads to signalling games.

Signalling Games

General Form

Definition 5 (Signalling game)

A **signalling game** is a tuple $\langle \Omega, \Theta_S, \Theta_H, P, \mathcal{F}, \mathcal{A}, u_S, u_H \rangle$ with:

1. Ω: A set of possible worlds.
2. Θ_S, Θ_H: two finite set of types for the speaker S and the hearer H.
3. P: a probability measure on $\Omega \times \Theta_S \times \Theta_H$;
4. $\mathcal{F}$: a set of signals from which the speaker S chooses his utterance.
5. $\mathcal{A}$: the set of actions from which the hearer H chooses his action.
6. u_S, u_H: payoff functions which map sequences $\langle v, F, a \rangle \in \Omega \times \mathcal{F} \times \mathcal{A}$ to real numbers.

Payoffs

1. The payoff functions u_S, u_H represent the **preferences** of the interlocutors over outcomes of their interaction.
2. We will mostly assume that the payoff functions are provided by a **joint** payoff function u.
3. That the payoff function is **joint** means that the preferences of speaker and hearer are identical.

$\Rightarrow$ It is a **games of pure coordination**!

Types

1. Types may be arbitrary objects. They don't have intrinsic meaning.
2. The information set of an agent is defined by an indiscernibility relation between tree nodes.
3. That we chose Ω for the hearer was for purely mnemotechnical reasons. We could have chosen any other object as well.
4. If the hearer's type is the same for all possibilities, then the hearer has no private knowledge.
5. In this case, we can simplify the game by eliminating the hearer's types.

Pure Strategies in a Signalling Game

1. Pure strategies are functions from information sets into action sets.
2. The speaker's information set is defined by his type θ_S.
3. The hearer's information set is defined by his type θ_H and the speaker's previous message $F \in \mathcal{F}$.

Pure Strategies:

$$S : \Theta_S \longrightarrow \mathcal{F} \text{ and } H : \Theta_H \times \mathcal{F} \longrightarrow \mathcal{A}.$$

Mixed Strategies in a Signalling Game

1. Mixed strategies are functions from information sets into the set of probability distributions over an action set.
2. The information sets do not change.

We write:

- $S(F|\theta_S)$: the probability with which the speaker sends the form F given type θ_S.
- $H(a|\theta_H, F)$: the probability with which the hearer chooses action a given type θ_H and message F.
- If there is only one hearer type, the hearer's mixed strategy is of the form $H(\,.\,|F)$.

Section 6

Parikh's Example: Resolving Ambiguities

[Parikh, 2001]

Resolving Ambiguities

Example 6 (Parikh's standard example)

1. Every ten minutes a man gets mugged in New York. (A)
2. Every ten minutes some man or other gets mugged in New York. (F)
3. Every ten minutes a particular man gets mugged in New York. (F')

- An utterance of A is ambiguous.
- F and F' are unambiguous alternatives for the two possible readings of A.
- Assume that the speaker says A. How to interpret it?

Meanings

The hearer has to interpret A. Possible meanings are:

1. φ: Meaning of 'every ten minutes some man or other gets mugged in New York.'
2. φ': Meaning of 'Every ten minutes a particular man gets mugged in New York.'

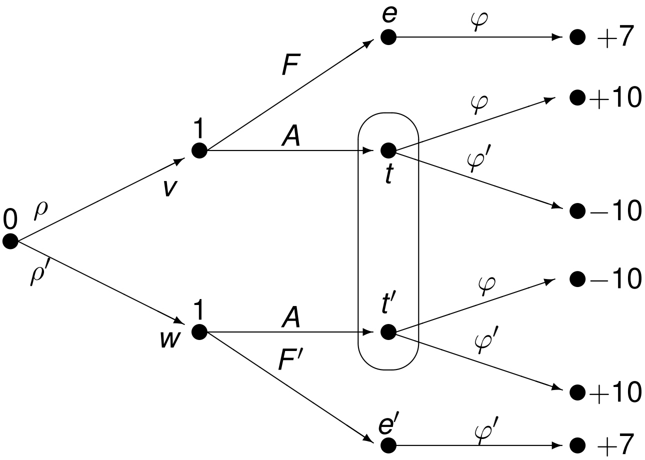
Possible speaker types are:

1. $\theta_s = v$: State where the speaker knows that φ.
2. $\theta'_s = w$: State where the speaker knows that φ'.

Probabilities:

1. ρ: Probability of v.
2. $\rho' = 1 - \rho$: Probability of w.

The Game Tree



The Strategies

	v	w
S	A	A
S'	A	F'
S''	F	A
S'''	F	F'

	A	F	F'
H	φ	φ	φ'
H'	φ'	φ	φ'

The Strategies

Speaker: $\{S, S', S'', S'''\}$

Hearer: $\{H, H'\}$

The Payoffs

v	H	H'
S	10	-10
S'	10	-10
S''	7	7
S'''	7	7

w	H	H'
S	-10	10
S'	7	7
S''	-10	10
S'''	7	7

The Payoffs
Left: In situation v
Right: In situation w

Expected Utilities

Case of pure strategies

1. **Assumption**: Rational players maximise their expected utilities.
2. Depends on the probability $P(v)$, the strategies S, H and payoffs.
3. **Speaker**:

$$\mathscr{E}_S(S|H) = \sum_v P(v)\, u_S(v, S(v), H(S(v))). \tag{6.2}$$

4. **Hearer**:

$$\mathscr{E}_H(H|S) = \sum_v P(v)\, u_H(v, S(v), H(S(v))). \tag{6.3}$$

5. In our example: $u_S = u_H$, and therefore $\mathscr{E}_S(S|H) = \mathscr{E}_H(H|S)$.

Expected Utilities

	H	H'
S	8	-8
S'	9.7	-8.3
S''	5.3	7.3
S'''	7	7

The Expected Payoffs

Probability of v: $\rho = 0.9$

Probability of w: $\rho' = 0.1$

Analysis

- There are two Nash equilibria: (S', H) and (S'', H').

	H	H'
S	8	-8
S'	9.7	-8.3
S''	5.3	7.3
S'''	7	7

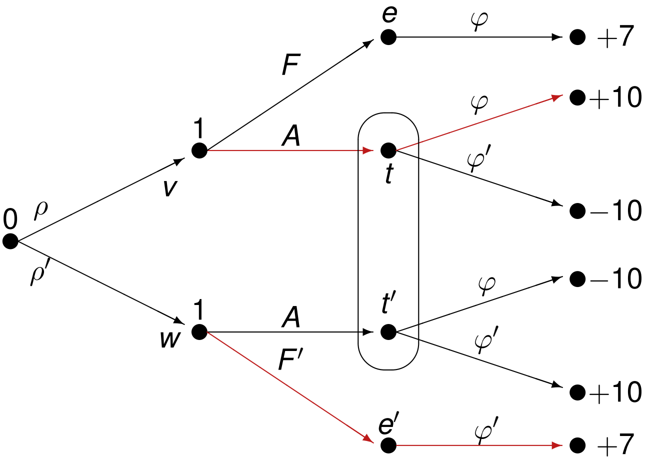
- The first one is also a Pareto Nash equilibrium.

	H	H'
S	8	-8
S'	9.7	-8.3
S''	5.3	7.3
S'''	7	7

- Assuming that rational players agree on the Pareto Nash equilibrium $\Rightarrow$ they will choose (S', H).
- With (S', H) the utterance A should be interpreted as meaning φ:

  A: Every ten minutes a man gets mugged in New York.
  φ: Every ten minutes some man or other gets mugged in New York.

The Pareto Nash Solution



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