Game Theoretic Pragmatics

Day 2

Basic Game Theory and the Iterated Best Response Model

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Course Outline

1. Introduction and Motivation
2. Basic Game Theory and the Iterated Best Response Model
3. The Basic Optimal Answer Model
4. Aspects of Bounded Rationality
5. Some Extensions of the Optimal Answer Model

Outline

Section 1

Basics of Game Theory

[Benz et al., 2006]

Game and Decision Theory

- Decision theory: Concerned with decisions of individual agents
- Game theory: Concerned with interdependent decisions of several agents.

Decision Problems and Conditional Probabilities

Decision Situations

- Take an umbrella with you when leaving the house.
- Choose between several candidates for a job.
- Decide where to look for a book which you want to buy.

Decision Problems

Definition 1 (Decision problem)

A decision problem is a triple $\langle(\Omega, P), \mathcal{A}, u\rangle$ such that

1. (Ω, P) is a finite discrete probability space;
2. $\mathcal{A}$ a non–empty set;
3. $u : \Omega \times \mathcal{A} \longrightarrow \mathbb{R}$ a function.

Terminology:

1. $\mathcal{A}$ is called the action set, and its elements actions.
2. u is called a payoff or utility function.

Example: Taking an Umbrella with you

1. Worlds (Ω):
   - w_1: rainy day.
   - w_2: cloudy but dry weather.
   - w_3: sunny day.
2. Probabilities (P): $P(w_1) = \frac{1}{4}$; $P(w_2) = \frac{1}{8}$; $P(w_3) = \frac{5}{8}$
3. Actions ($\mathcal{A}$): a: taking umbrella with you; b: taking no umbrella.
4. Utilities (u):
   - rainy day: $u(w_1, a) = 1$, $u(w_1, b) = -1$.
   - cloudy day: $u(w_2, a) = -1$, $u(w_2, b) = 0$.
   - sunny day: $u(w_3, a) = -1$, $u(w_3, b) = 0$.

Decision Criterion

- It is assumed that rational agents are Bayesian utility maximisers.
- If an agent chooses an action, then the action's expected utility must be maximal.

Definition 2 (Expected Utility)

Let $\langle(\Omega, P), \mathcal{A}, u\rangle$ be a decision problem and $a \in \mathcal{A}$ an action. The expected utility of a is defined by:

$$EU(a) = \sum_{v \in \Omega} P(v) \cdot u(v, a) \tag{1.1}$$

Optimising expected utilities means that a decision maker will choose an action a only if

$$EU(a) = \max_{b \in \mathcal{A}} EU(b). \tag{1.2}$$

Example: Taking an Umbrella with you

1. Worlds: w_1: rainy, w_2: cloudy but dry, w_3: sunny.
2. Utilities (u) of actions:
   - a take umbrella: $u(w_1, a) = 1$, $u(w_2, a) = -1$, $u(w_3, a) = -1$;
   - b take no umbrella: $u(w_1, b) = -1$, $u(w_2, b) = 0$, $u(w_3, b) = 0$.
3. Probabilities (P): $P(w_1) = \frac{1}{4}$; $P(w_2) = \frac{1}{8}$; $P(w_3) = \frac{5}{8}$

Expected utilities of actions:

$$\begin{aligned} EU(\text{taking umbrella}) &= \frac{1}{4} \cdot 1 - \frac{1}{8} \cdot 1 - \frac{5}{8} \cdot 1 \\ &= \frac{2}{8} - \frac{6}{8} = -\frac{1}{2} \\ EU(\text{taking no umbrella}) &= -\frac{1}{4} \cdot 1 + \frac{1}{8} \cdot 0 + \frac{5}{8} \cdot 0 = -\frac{1}{4} + 0 = -\frac{1}{4} \end{aligned}$$

$\Rightarrow$ Rational agent decides for taking no umbrella.

Learning

How are expectations changed by new information?

Example 3

Before John looked out of window:

$$P(\text{cloudy} \cap \text{will-rain}) = \frac{1}{4}; P(\text{cloudy}) = \frac{3}{8}.$$

Looking out of window John learns that it is cloudy.
$\Rightarrow$ What is the new probability of 'will-rain'?

Need conditional probabilities!

Conditional Probabilities

Definition 4

Let (Ω, P) be a discrete probability space representing expectations prior to new observation A. For any hypothesis B the conditional probability after learning A is defined as:

$$P(B|A) = P(B \cap A)/P(A) \text{ for } P(A) > 0 \tag{1.3}$$

For $P(A) = 0$, $P(B|A)$ is not defined!

Example

- Before John looked out of window:

$$P(\text{cloudy} \cap \text{will-rain}) = \frac{1}{4}; \quad P(\text{cloudy}) = \frac{3}{8}.$$

- After John learns that it is cloudy:

$$\begin{aligned} P(\text{will-rain}|\text{cloudy}) &= P(\text{cloudy} \cap \text{will-rain})/P(\text{cloudy}) \\ &= \frac{1}{4} : \frac{3}{8} = \frac{2}{3}. \end{aligned}$$

- Expected utilities after learning that it is cloudy:

$$\begin{aligned} EU(\text{taking umbrella}|\text{cloudy}) &= \frac{2}{3} \cdot 1 - \frac{1}{3} \cdot 1 = \frac{1}{3} \\ EU(\text{taking no umbrella}) &= -\frac{2}{3} \cdot 1 + \frac{1}{3} \cdot 0 = -\frac{2}{3} \end{aligned}$$

$\Rightarrow$ Rational agent decides for taking umbrella.

Signalling Games

Signalling Games

General Form

Definition 5 (Signalling game)

A **signalling game** is a tuple $\langle \Omega, \Theta_S, \Theta_H, P, \mathcal{F}, \mathcal{A}, u_S, u_H \rangle$ with:

1. Ω: A set of possible worlds.
2. Θ_S, Θ_H: two finite set of types for the speaker S and the hearer H.
3. P: a probability measure on $\Omega \times \Theta_S \times \Theta_H$;
4. $\mathcal{F}$: a set of signals from which the speaker S chooses his utterance.
5. $\mathcal{A}$: the set of actions from which the hearer H chooses his action.
6. u_S, u_H: payoff functions which map sequences $\langle v, F, a \rangle \in \Omega \times \mathcal{F} \times \mathcal{A}$ to real numbers.
7. Furthermore: We assume that all sets are finite!

Probablilities

1. $P(v, \theta_S, \theta_H)$: Probability with which nature chooses world v, speaker type θ_S, and hearer type θ_H.
2. We can think of $P(v, \theta_S, \theta_H)$ as the result of first choosing v, and then simultaneously θ_S and θ_H:
3. From $P(v, \theta_S, \theta_H)$ the probability $P(v)$ of a state of the world can be calculated by:

$$P(v) = \sum_{\theta_S, \theta_H} P(v, \theta_S, \theta_H). \tag{1.4}$$

Pure Strategies in a Signalling Game

1. Pure strategies are functions from information sets into action sets.
2. The speaker's information set is defined by his type θ_S.
3. The hearer's information set is defined by his type θ_H and the speaker's previous message $F \in \mathcal{F}$.

Pure Strategies:

$$S : \Theta_S \longrightarrow \mathcal{F} \text{ and } H : \Theta_H \times \mathcal{F} \longrightarrow \mathcal{A}.$$

Expected Utilities

Expected Utilities

1. What is the expected utility of an act given uncertainty about the state of the world?
2. **Speaker**: What is the expected utility of sending a linguistic form *F* if the hearer is known to choose *a*?
3. **First case:** No private knowledge involved:

$$\mathcal{E}_S(F|a) = \sum_v P(v)\, u_S(v, F, a). \tag{1.5}$$

4. **Hearer**: What is the expected utility of choosing an act *a* if the speaker has sent form *F*?

$$\mathcal{E}_H(a|F) = \sum_v P(v)\, u_H(v, F, a). \tag{1.6}$$

Expected Utilities

Case of pure strategies

1. **Second case:** The speaker knows the actual state of the world, and the hearer has no private knowledge.
2. Depends on the probability $P(v)$, the strategies S, H and payoffs.
3. **Speaker**:

$$\mathscr{E}_S(S|H) = \sum_v P(v)\, u_S(v, S(v), H(S(v))). \tag{1.7}$$

4. **Hearer**:

$$\mathscr{E}_H(H|S) = \sum_v P(v)\, u_H(v, S(v), H(S(v))). \tag{1.8}$$

5. If $u_S = u_H$, then $\mathscr{E}_S(S|H) = \mathscr{E}_H(H|S)$.

Expected Utilities

Case of pure strategies — general case

1. If the speaker does not know the actual state of the world, or if the hearer has private knowledge, then the probabilities of their respective types have to be taken into consideration.

$\Rightarrow$ **General case:** The speaker may not know the actual state of the world, and the hearer may have private knowledge.

2. Depends on the probability $P(v, \theta_S, \theta_H)$, the strategies S, H and payoffs.

Expected Utilities

Case of pure strategies — general case

1. **Speaker**:

$$\mathscr{E}_S(S|H) = \sum_{(v,\theta_S,\theta_H)} P(v,\theta_S,\theta_H)\, u_S(v, S(\theta_S), H(S(\theta_S))). \tag{1.9}$$

2. **Hearer**:

$$\mathscr{E}_H(S|H) = \sum_{(v,\theta_S,\theta_H)} P(v,\theta_S,\theta_H)\, u_H(v, S(\theta_S), H(S(\theta_S))). \tag{1.10}$$

3. If $u_S = u_H$, then $\mathscr{E}_S(S|H) = \mathscr{E}_H(H|S)$.

Mixed Strategies in a Signalling Game

1. Mixed strategies are functions from information sets into the set of probability distributions over an action set.
2. The information sets do not change.

We write:

- $S(F|\theta_S)$: the probability with which the speaker sends the form F given type θ_S.
- $H(a|\theta_H, F)$: the probability with which the hearer chooses action a given type θ_H and message F.

- If there is only one hearer type, the hearer's mixed strategy is of the form $H(\,.\,|F)$.

Expected Utilities

Case of mixed strategies

1. **Assumption**: Rational players maximise the expected utility of their strategies.
2. Depends on the probability $P(v)$, the strategies S, H and payoffs.
3. **Speaker**: $\mathscr{E}_S(S|H)$ is defined by
$$\sum_{(v,\theta_S,\theta_H)} P(v,\theta_S,\theta_H) \sum_{F\in\mathcal{F}} S(F|\theta_S) \sum_{a\in\mathcal{A}} H(a|\theta_H, F)\, u_S(v, F, a). \quad (1.11)$$
4. **Hearer**: $\mathscr{E}_S(S|H)$ is defined by
$$\sum_{(v,\theta_S,\theta_H)} P(v,\theta_S,\theta_H) \sum_{F\in\mathcal{F}} S(F|\theta_S) \sum_{a\in\mathcal{A}} H(a|\theta_H, F)\, u_H(v, F, a). \quad (1.12)$$
5. If $u_S = u_H$, then $\mathscr{E}_S(S|H) = \mathscr{E}_H(H|S)$.

Conditional Expected Utilities

1. What is the expected utility of sending a form F given that the speaker has type θ_S and the hearer follows strategy H?

$$\mathscr{E}_S(F|H,\theta_S) = \sum_{v,\theta_H} P(v,\theta_H|\theta_S) \sum_{a\in\mathcal{A}} H(a|\theta_H,F)\, u_S(v,F,a). \quad (1.13)$$

2. What is the expected utility of choosing act a given that the hearer has type θ_S, the speaker follows strategy S, and the speaker sent form F?

$$\mathscr{E}_H(a|S,F,\theta_H) = \sum_{v} P(v|\theta_H,F)\, u_H(v,F,a). \quad (1.14)$$

With:

$$P(v|\theta_H,F) = \frac{\sum_{\theta_S} P(v,\theta_S,\theta_H)\, S(F|\theta_S)}{\sum_v \sum_{\theta_S} P(v,\theta_S,\theta_H)\, S(F|\theta_S)}. \quad (1.15)$$

Best Response

Best Response

1. Given: Speaker knows that hearer follows strategy H.
2. If $\mathscr{E}_S(S_1|H) < \mathscr{E}_S(S_2|H)$ then maximising expected utilities means that speaker will **not** choose strategy S_1.
3. Speaker will choose from the set

$$\mathscr{B}_S(H) := \{S \mid \forall S' \, \mathscr{E}_S(S'|H) \leq \mathscr{E}_S(S|H)\} \tag{1.16}$$

4. If $S \in \mathscr{B}_S(H)$, then S is called a best response to H, and $\mathscr{B}_S(H)$ the set of best responses to H.

5. Analogously, the set of the hearer's best responses to a strategy S is defined by

$$\mathscr{B}_H(S) := \{H \mid \forall H' \, \mathscr{E}_H(H'|S) \leq \mathscr{E}_H(H|S)\} \tag{1.17}$$

Best Response

- For calculating best responses to acts, we need to take the agents information sets into account.

1. Speaker's best responses to strategy H given his type θ_S:

$$\mathscr{B}_S(H, \theta_S) = \{F \in \mathcal{F} \mid \forall F' \ \mathscr{E}_S(F'|H, \theta_S) \leq \mathscr{E}_S(F|H, \theta_S)\}. \quad (1.18)$$

2. Hearer's best responses to a signal F given type θ_H and speaker strategy S:

$$\mathscr{B}_H(F, S, \theta_H) = \{a \in \mathcal{A} \mid \forall a' \ \mathscr{E}_H(a'|S, \theta_H, F) \leq \mathscr{E}_H(a|S, \theta_H, F)\}. \quad (1.19)$$

Canonical Best Response

- It is convenient to have a canonical unique best response to the strategy of the other interlocutor.
- Let (S, H) be a pair of signalling strategies:

1. S is the speaker's canonical best response to strategy H if for each type θ_s S assigns equal probability to the elements of:

$$\mathscr{B}_S(H, \theta_s) = \{F \in \mathcal{F} \mid \forall F' \; \mathscr{E}_S(F'|H, \theta_s) \leq \mathscr{E}_S(F|H, \theta_s)\}. \qquad (1.20)$$

2. H is the hearer's canonical best response to strategy S if for each type θ_s and form F H assigns equal probability to the elements of:

$$\mathscr{B}_H(F, S, \theta_H) = \{a \in \mathcal{A} \mid \forall a' \; \mathscr{E}_H(a'|S, \theta_H, F) \leq \mathscr{E}_H(a|S, \theta_H, F)\}. \qquad (1.21)$$

Some Standard Equilibirium Concepts

Nash, Pareto

Nash Equilibria

Definition 6

A strategy pair (S, H) is a (weak) Nash equilibrium of a signalling game $\mathscr{G}$, iff S is a best response to H, and H a best response to S:

$$S \in \mathscr{B}_S(H) \ \text{ and } \ H \in \mathscr{B}_H(S). \tag{1.22}$$

A strategy pair (S, H) is a strict Nash equilibrium, iff

$$\mathscr{B}_S(H) = \{S\} \ \text{ and } \ \mathscr{B}_H(S) = \{H\}. \tag{1.23}$$

Pareto Nash Equilibria

Definition 7

A strategy pair (S, H) is a (weak) Pareto Nash equilibrium of a signalling game $\mathscr{G}$, iff (S, H) is Nash equilibrium for which there is no other Nash equilibrium (S', H') for which:

$$\mathscr{E}_S(S|H) < \mathscr{E}_S(S'|H') \text{ and } \mathscr{E}_H(H|S) < \mathscr{E}_H(H'|S'). \tag{1.24}$$

A strategy pair (S, H) is a strict Pareto Nash equilibrium, iff (S, H) is Nash equilibrium for which there is no other Nash equilibrium (S', H') for which:

$$\mathscr{E}_S(S|H) \leq \mathscr{E}_S(S'|H') \text{ and } \mathscr{E}_H(H|S) < \mathscr{E}_H(H'|S').$$

or

$$\mathscr{E}_S(S|H) < \mathscr{E}_S(S'|H') \text{ and } \mathscr{E}_H(H|S) \leq \mathscr{E}_H(H'|S').$$

Application: Parikh's Example

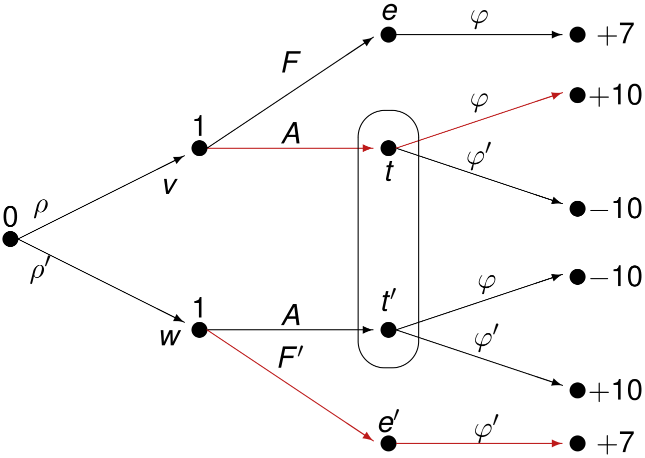
Resolving Ambiguities

Example 8 (Parikh's standard example)

1. Every ten minutes a man gets mugged in New York. (A)
2. Every ten minutes some man or other gets mugged in New York. (F)
3. Every ten minutes a particular man gets mugged in New York. (F')

- An utterance of A is ambiguous.
- F and F' are unambiguous alternatives for the two possible readings of A.

The Pareto Nash Solution



Analysis

- There are two Nash equilibria: (S', H) and (S'', H').

	H	H'
S	8	-8
S'	9.7	-8.3
S''	5.3	7.3
S'''	7	7

- The first one is also a Pareto Nash equilibrium.

	H	H'
S	8	-8
S'	9.7	-8.3
S''	5.3	7.3
S'''	7	7

Section 2

Iterated Best Response Models

[Franke, 2009, Jäger and Ebert, 2009]

Bounded Rationality

1. **Classical game theory**: Rationality assumptions are often seen as being unrealistically strong:
   i. Common knowledge of game structure;
   ii. Logical omniscience.
2. **Evolutionary game theory**: Radical counter position:
   i. No reasoning capabilities;
   ii. Strategies spread by replication.

   May be seen as unrealistically weak!

$\Rightarrow$ Search for more realistic models!

p–Beauty Contest

[Camerer, 2003][Franke, 2009, p. 49f]

Example 9

- There are $n \geq 2$ players.
- Each player has to choose a number in $\{0, 1, 2, \ldots, 100\}$.
- The player who is closest to $\frac{2}{3}$rd of the average wins.

- This game has two Nash equilibria: everyone choosing 0 and everyone choosing 1.
- Experiments show that people choose between 20 and 30.

p–Beauty Contest
Reasoning

Explaining the Nash equilibrium:

1. If players choose at random, then the expected average is 50.
2. In order to come close to $\frac{2}{3}$ of average choose 33.
   (**best response!**)
   New expected average: 33.
3. In order to come close to $\frac{2}{3}$ of average choose 22.
   New expected average: 22.
4. In order to come close to $\frac{2}{3}$ of average choose 15.
   New expected average: 15.
5. In order to come close to $\frac{2}{3}$ of average choose 10.
   New expected average: 10.
6. …

1. Players arrive at Nash equilibrium via an iterated reasoning process.
2. Each iteration step involves change of perspective and reasoning about other players.
3. Nash equilibrium obviously not suitable for describing actual behaviour.
4. **Bounded rationality**: Players perform some of the reasoning steps, but do not drive reasoning to its limits.
5. Transferring this idea to pragmatics leads to **Iterated Best Response** model.

Iterated Best Response Model

Explanation of Implicatures, [Jäger and Ebert, 2009, Franke, 2009]

Iterated Best Response Model (hearer line):

1. Start with a signalling game $\mathcal{G}$ in which the hearer makes his choice on the basis of literal meaning (H^0).
2. Calculate the speaker's best response S^1 to H^0.
3. Calculate the hearer's best response H^2 to S^1.
4. Iterate this process until it stabilises in a strategy pair $(S, H) = (S^n, H^{n\pm1})$
5. Implicature $F +> Q$ is explained if:

$$H(F) \Rightarrow Q. \tag{2.25}$$

Iterated Best Response Model

Scalar Implicatures, [Franke, 2009]

Example 10

Some of the boys came to the party. $+>$ not all came

$$H_0 = \left\{ \begin{array}{l} \textit{some} \mapsto \theta_{\exists\neg\forall}, \theta_{\forall} \\ \textit{all} \mapsto \theta_{\forall} \end{array} \right\} \quad (2.26)$$

$$S_1 = \left\{ \begin{array}{l} \theta_{\exists\neg\forall} \mapsto \textit{some} \\ \theta_{\forall} \mapsto \textit{all} \end{array} \right\} \quad (2.27)$$

$$H_2 = \left\{ \begin{array}{l} \textit{some} \mapsto \theta_{\exists\neg\forall} \\ \textit{all} \mapsto \theta_{\forall} \end{array} \right\} \quad (2.28)$$

$$S_3 = S_1 \quad (2.29)$$

$$H_4 = H_2. \quad (2.30)$$

Iterated Best Response Model

Speaker line

Iterated Best Response Model (speaker line):

1. Start with a signalling game $\mathcal{G}$ in which the speaker arbitrarily sends true signals (S^0).
2. Calculate the hearer's best response H^1 to S^0.
3. Calculate the speaker's best response S^2 to H^1.
4. Iterate this process until it stabilises in a strategy pair $(S, H) = (S^n, H^{n\pm 1})$

Iterated Best Response Model

Speaker line

$$S_0 = \left\{ \begin{array}{l} \theta_{\exists\neg\forall} \mapsto \textit{some} \\ \theta_{\forall} \mapsto \textit{some}, \textit{all} \end{array} \right\}$$

$$H_1 = \left\{ \begin{array}{l} \textit{some} \mapsto \theta_{\exists\neg\forall}, \theta_{\forall} \\ \textit{all} \mapsto \theta_{\forall} \end{array} \right\}$$

$$S_2 = \left\{ \begin{array}{l} \theta_{\exists\neg\forall} \mapsto \textit{some} \\ \theta_{\forall} \mapsto \textit{all} \end{array} \right\}$$

$$H_3 = \left\{ \begin{array}{l} \textit{some} \mapsto \theta_{\exists\neg\forall} \\ \textit{all} \mapsto \theta_{\forall} \end{array} \right\}$$

$$S_4 = S_2$$

$$H_5 = H_3.$$

Iterated Best Response Model

Speaker line

Example 11

Some of the boys came to the party. $+>$ not all came

1. Hearer line: $H_4 = H_2$ and $S_3 = S_1$.
2. Speaker line: $H_5 = H_3$ and $S_4 = S_2$.
3. And: $H_5 = H_4 = H_3 = H_2$ and $S_4 = S_3 = S_2 = S_1$.
4. Hence, (S_1, H_2) is stable.

The Iterated Best Response (IBR) Model

[Franke, 2009, p. 57]

$$
\begin{tikzcd}
\text{sends any true message} \arrow[r] & S_0 & & H_0 & \text{interprets message literally} \arrow[l] \\
\text{best response to } S_0 \arrow[r] & H_1 \arrow[u, Rightarrow] & & S_1 \arrow[u, Rightarrow] & \text{best response to } R_0 \arrow[l] \\
\text{best response to } R_1 \arrow[r] & S_2 \arrow[u, Rightarrow] & & H_2 \arrow[u, Rightarrow] & \text{best response to } S_1 \arrow[l] \\
\vdots & \vdots & & \vdots & \vdots
\end{tikzcd}
$$

Iterated Best Response Strategies

The speaker's strategies

For a finite set X let $\overline{X}$ be the set of all probability distributions which assign equal probability to the elements of X.

1. Let $\mathscr{S}_0$ be the set of all speaker strategies for which the speaker only sends true signals:

$$\mathscr{S}_0 = \{S \mid \forall \theta_s\, S(\,.\,|\theta_s) \in \overline{\{F \mid P([\![F]\!]|\theta_s) = 1\}}\} \tag{2.31}$$

2. Recursion step: Let $\mathscr{S}_n$ and $\mathscr{H}_n$ be given, then:

$$\mathscr{S}_{n+1} = \{S \mid \exists H \in \mathscr{H}_n\, \forall \theta_s\colon S(\,.\,|\theta_s) \in \overline{\mathscr{B}_S(H, \theta_s)}\}. \tag{2.32}$$

Iterated Best Response Strategies

The hearer's strategies

For a finite set X let $\overline{X}$ be the set of all probability distributions which assign equal probability to the elements of X.

1. Let $\mathscr{H}_0$ be the set of all hearer strategies for which the hearer interprets forms semantically:

$$\mathscr{H}_0 = \{H \mid \forall\theta_H, F\colon H(\,.\,|\theta_H) \in \overline{\mathscr{B}_H([\![F]\!], \theta_H)}\} \tag{2.33}$$

2. Recursion step: Let $\mathscr{S}_n$ and $\mathscr{H}_n$ be given, then let:

$$\mathscr{H}_{n+1} = \{H \in \mathscr{H}_n \mid \exists S \in \mathscr{S}_n\, \forall\theta_H, F\colon H(\,.\,|\theta_H) \in \overline{B}\}. \tag{2.34}$$

with

$$B = \begin{cases} \mathscr{B}_H(S, F, \theta_H) \text{ if } \exists v, \theta_S\, P(v, \theta_S, \theta_H)\, S(F|\theta_S) > 0, \\ \mathscr{B}_H([\![F]\!], \theta_H) \text{ if } \neg\exists v, \theta_S\, P(v, \theta_S, \theta_H)\, S(F|\theta_S) > 0, \end{cases} \tag{2.35}$$

Iterated Best Response Limit Prediction

The hearer's strategies

1. The IBR strategies defined before are in general not in equilibrium.
2. A equilibrium concept can be derived if one considers the strategies which occur in infinitely many levels:

$$\mathscr{S}^* = \{S \mid \forall n \, \exists m > n \; S \in \mathscr{S}_m\}. \tag{2.36}$$

$$\mathscr{H}^* = \{H \mid \forall n \, \exists m > n \; H \in \mathscr{H}_m\}. \tag{2.37}$$

Advantages

1. Reasoning about each other becomes explicit.
2. Allows to measure number of reasoning steps for reaching stability.
3. Starts out from pre–defined semantics, and shows how it is affected by iterated optimisation of strategies.

Interpretation

- Michael Franke looks at the Iterated Best Respons model as a competence model.
- This means that a stable strategy pair is the result of repeated social interaction with other interlocutors.
- It is not an online model.
- Having encountered a certain type of utterance situation again and again, the competence for behaving optimally in this situation is gradually acquired.

Section 3

Applications of the Iterated Best Response Model

[Franke, 2009]

Multiple Scalar Items

[Franke, 2009, p. 134]

Example 12 ([Sauerland, 2004])

1. $F_{some|some}$: Kai ate some of the strawberries and Hannes ate some of the carrots.
2. $+>$ It's not the case that Kai ate all of the strawberries and Hannes ate some of the carrots.
3. $+>$ It's not the case that Kai ate some of the strawberries and Hannes ate all of the carrots.
4. $+>$ It's not the case that Kai ate all of the strawberries and Hannes ate all of the carrots.

Alternative Forms

Assumption: When producing $F_{some|some}$, the speaker had to make a choice between $F_{some|some}$ and the following forms:

1. $F_{some|all}$: Kai ate some of the strawberries and Hannes ate all of the carrots.
2. $F_{all|some}$: Kai ate all of the strawberries and Hannes ate some of the carrots.
3. $F_{all|all}$: Kai ate all of the strawberries and Hannes ate all of the carrots.

Interpretation games (Franke)

Definition 13 (Interpretation games)

An interpretation game is an interpreted signalling game $\mathscr{G} = \langle \Omega, \Theta_S, P, \mathcal{F}, \mathcal{A}, u, [\![\,.\,]\!] \rangle$ for which

1. The hearer has no private information.
2. It is a game of pure coordination, i.e. $u_S = u_H$.
3. $\Omega = \Theta_S$: the speaker's type and the state of the world are identified with each other.
4. Θ_S is finite, and P is a **flat** prior probability, i.e. the types have equal probability.
5. $\mathcal{A} = \Theta_S$: the hearer's task is do identify the speaker's type.
6. Hence:

$$u_H(\theta_S, F, \theta'_S) = \begin{cases} 1 \text{ if } \theta_S = \theta'_S, \\ 0 \text{ if } \theta_S \neq \theta'_S, \end{cases} . \tag{3.38}$$

Construction Rules for Franke's IBR Models

1. The game is constructed on the basis of the form F produced by the speaker.
2. $\mathcal{F}$, i.e. the speaker's forms, are the sets of alternatives which must be given independently.
3. In case of scalar expressions, these alternatives are the result of replacing critical expression by their stronger scalar alternatives.
4. The speakers type are defined as follows:
   i. Let F be the form chosen by the speaker, and $F_1, \ldots, F_n$ its alternatives.
   ii. Let $G_i \in \{F_i, \neg F_i\}$.

   Then the types are defined by all **consistent conjunctions** of the form:

$$F \wedge G_1 \wedge \ldots \wedge G_n. \tag{3.39}$$

The Definition of Types for Sauerland Example

Example: $\theta_{\forall|\exists\neg\forall}$ is defined as the set of all worlds which satisfy:

$$F_{some|some} \wedge \neg F_{some|all} \wedge F_{all|some} \wedge \neg F_{all|all}.$$

This leads to the following set of types:

	$F_{some\|all}$	$F_{all\|some}$	$F_{all\|all}$
$\theta_{\forall\|\forall}$	1	1	1
$\perp$	1	1	0
$\perp$	1	0	1
$\theta_{\exists\neg\forall\|\forall}$	1	0	0
$\perp$	0	1	1
$\theta_{\forall\|\exists\neg\forall}$	0	1	0
$\perp$	0	0	1
$\theta_{\exists\neg\forall\|\exists\neg\forall}$	0	0	0

$\perp$: is the inconsistent type, and is not realised; i.e. $P(\perp) = 0$.

The IBR Model

	$P(\theta_s)$	$\theta_{\forall\|\forall}$	$\theta_{\exists\neg\forall\|\forall}$	$\theta_{\forall\|\exists\neg\forall}$	$\theta_{\exists\neg\forall\|\exists\neg\forall}$
$\theta_{\forall\|\forall}$	$1/4$	1,1	0,0	0,0	0,0
$\theta_{\exists\neg\forall\|\forall}$	$1/4$	0,0	1,1	0,0	0,0
$\theta_{\forall\|\exists\neg\forall}$	$1/4$	0,0	0,0	1,1	0,0
$\theta_{\exists\neg\forall\|\exists\neg\forall}$	$1/4$	0,0	0,0	0,0	1,1

	$F_{some\|some}$	$F_{some\|all}$	$F_{all\|some}$	$F_{all\|all}$
$\theta_{\forall\|\forall}$	1	1	1	1
$\theta_{\exists\neg\forall\|\forall}$	1	1	0	0
$\theta_{\forall\|\exists\neg\forall}$	1	0	1	0
$\theta_{\exists\neg\forall\|\exists\neg\forall}$	1	0	0	0

The IBR Sequence

Hearer Line

$$H_0 = \left\{ \begin{array}{lll} F_{some|some} & \mapsto & \theta_{\exists\neg\forall|\exists\neg\forall}, \theta_{\exists\neg\forall|\forall}, \theta_{\forall|\exists\neg\forall}, \theta_{\forall|\forall} \\ F_{some|all} & \mapsto & \theta_{\exists\neg\forall|\forall}, \theta_{\forall|\forall} \\ F_{all|some} & \mapsto & \theta_{\forall|\exists\neg\forall}, \theta_{\forall|\forall} \\ F_{all|all} & \mapsto & \theta_{\forall|\forall} \end{array} \right\}$$

$$S_1 = \left\{ \begin{array}{lll} \theta_{\exists\neg\forall|\exists\neg\forall} & \mapsto & F_{some|some} \\ \theta_{\exists\neg\forall|\forall} & \mapsto & F_{some|all} \\ \theta_{\forall|\exists\neg\forall} & \mapsto & F_{all|some} \\ \theta_{\forall|\forall} & \mapsto & F_{all|all} \end{array} \right\}$$

The IBR Sequence

The hearer's limit strategy

It is immediately clear that (S_1, H_2) is stable:

$$H_2 = \left\{ \begin{array}{lcl} F_{some|some} & \mapsto & \theta_{\exists\neg\forall|\exists\neg\forall} \\ F_{some|all} & \mapsto & \theta_{\exists\neg\forall|\forall} \\ F_{all|some} & \mapsto & \theta_{\forall|\exists\neg\forall} \\ F_{all|all} & \mapsto & \theta_{\forall|\forall} \end{array} \right\} \qquad (3.40)$$

$\Rightarrow$ H_2 is predicted to hold in the limit.

- **Assumption:** The hearer's limit strategy determines the pragmatic meaning of a form!

An Example with Nested Quantifier Scope

[Franke, 2009, p. 136f]

Multiple Scalar Items

[Franke, 2009, p. 134]

Example 14 ([Sauerland, 2004])

1. $F_{some|some}$: Some of the students read some of the books.
2. $+>$ It's not the case that all of the students read some of the books.
3. $+>$ It's not the case that some of the students read all of the books.
4. $+>$ It's not the case that all of the students read all of the books.

Alternative Forms

Assumption: When producing $F_{some|some}$, the speaker had to make a choice between $F_{some|some}$ and the following forms:

1. $F_{some|all}$: Some of the students read all of the books.
2. $F_{all|some}$: All of the students read some of the books.
3. $F_{all|all}$: All of the students read all of the books.

Rule: The possible speaker types are constructed from the set of of the conjunctions of $F_{some|some}$ with the negated or un-negated alternatives.

The Definition of Types

1. **Remember**: The possible speaker types are constructed from the set of of the conjunctions of $F_{some|some}$ with the negated or un-negated alternatives.
2. We write $\exists^!$ for $\exists\neg\forall$.

	$F_{some\|all}$	$F_{all\|some}$	$F_{all\|all}$
$\theta_{\forall\forall}$	1	1	1
$\theta_{\forall\exists^!\wedge\exists^!\forall}$	1	1	0
$\perp$	1	0	1
$\theta_{\exists^!\forall}$	1	0	0
$\perp$	0	1	1
$\theta_{\forall\exists^!}$	0	1	0
$\perp$	0	0	1
$\theta_{\exists^!\exists^!}$	0	0	0

$\perp$: is the inconsistent type, not realised; i.e. $P(\perp) = 0$.

The IBR Model

	$P(\theta_s)$	$\theta_{\forall\forall}$	$\theta_{\forall\exists!\wedge\exists!\forall}$	$\theta_{\exists!\forall}$	$\theta_{\forall\exists!}$	$\theta_{\exists!\exists!}$
$\theta_{\forall\forall}$	$1/5$	1,1	0,0	0,0	0,0	0,0
$\theta_{\forall\exists!\wedge\exists!\forall}$	$1/5$	1,1	0,0	0,0	0,0	0,0
$\theta_{\exists!\forall}$	$1/5$	0,0	1,1	0,0	0,0	0,0
$\theta_{\forall\exists!}$	$1/5$	0,0	0,0	0,0	1,1	0,0
$\theta_{\exists!\exists!}$	$1/5$	0,0	0,0	0,0	0,0	1,1

	$F_{some\|some}$	$F_{some\|all}$	$F_{all\|some}$	$F_{all\|all}$
$\theta_{\forall\forall}$	1	1	1	1
$\theta_{\forall\exists!\wedge\exists!\forall}$	1	1	1	0
$\theta_{\exists!\forall}$	1	1	0	0
$\theta_{\forall\exists!}$	1	0	1	0
$\theta_{\exists!\exists!}$	1	0	0	0

The IBR Sequence

Hearer Line

$$H_0 = \left\{ \begin{array}{lcl} F_{all|all} & \mapsto & \theta_{\forall\forall} \\ F_{all|some} & \mapsto & \theta_{\forall\forall}, \theta_{\forall\exists! \wedge \exists!\forall}, \theta_{\forall\exists!} \\ F_{some|all} & \mapsto & \theta_{\forall\forall}, \theta_{\forall\exists! \wedge \exists!\forall}, \theta_{\exists!\forall} \\ F_{some|some} & \mapsto & \theta_{\forall\forall}, \theta_{\forall\exists! \wedge \exists!\forall}, \theta_{\exists!\forall}, \theta_{\forall\exists!}, \theta_{\exists!\exists!} \end{array} \right\}$$

$$S_1 = \left\{ \begin{array}{lcl} \theta_{\forall\forall} & \mapsto & F_{all|all} \\ \theta_{\forall\exists! \wedge \exists!\forall} & \mapsto & F_{some|all}, F_{all|some} \\ \theta_{\exists!\forall} & \mapsto & F_{all|some} \\ \theta_{\forall\exists!} & \mapsto & F_{all|some} \\ \theta_{\exists!\exists!} & \mapsto & F_{some|some} \end{array} \right\}$$

The IBR Sequence

Hearer Line

$$H_2 = \left\{ \begin{array}{lcl} F_{all|all} & \mapsto & \theta_{\forall\forall} \\ F_{all|some} & \mapsto & \theta_{\forall\exists!} \\ F_{some|all} & \mapsto & \theta_{\exists!\forall} \\ F_{some|some} & \mapsto & \theta_{\exists!\exists!} \end{array} \right\}$$

$$S_3 = \left\{ \begin{array}{lcl} \theta_{\forall\forall} & \mapsto & F_{all|all} \\ \theta_{\forall\exists!\wedge\exists!\forall} & \mapsto & F_{some|all}, F_{all|some}, F_{some|some} \\ \theta_{\exists!\forall} & \mapsto & F_{all|some} \\ \theta_{\forall\exists!} & \mapsto & F_{all|some} \\ \theta_{\exists!\exists!} & \mapsto & F_{some|some} \end{array} \right\}$$

- It is then easy to see that $H_4 = H_2$ and $S_5 = S_3$.

$\Rightarrow$ (S_3, H_2) is a stable limit signalling pair.

Remarks on Limit Strategy

1. $\theta_{\forall\exists!\wedge\exists!\forall}$ cannot be expressed by the limit strategy.
2. The conditional probability of $\theta_{\forall\exists!\wedge\exists!\forall}$ given any of the forms $F_{some|some}$, $F_{some|all}$, $F_{all|some}$ is still $1/4$.

$\Rightarrow$ Employs the same disambiguation rule as Parikh, i.e. the hearer disambiguates by choosing the more probable alternative.

3. The implicature is explained by the hearer's limit strategy!!

Multiple Scalar Items

[Franke, 2009, p. 134]

The hearer's limit strategy H_2 predicts:

- $F_{some|some}$: Some of the students read some of the books.
  has meaning
- $\theta_{\exists!\exists!}$: Some but not all of the students read some but not all of the books.

This means $F_{some|some}$ implicates:

1. It's not the case that all of the students read some of the books.
2. It's not the case that some of the students read all of the books.
3. It's not the case that all of the students read all of the books.

Iterated Best Response Model

Explanation of Implicatures, Summary

Iterated Best Response Model:

1. Start with a signalling game $\mathcal{G}$ in which the hearer makes his choice on the basis of literal meaning (H^0).
2. Calculate the speaker's best response S^1 to H^0.
3. Calculate the hearer's best response H^2 to S^1.
4. Iterate this process until it stabilises in a strategy pair $(S, H) = (S^n, H^{n\pm1})$
5. Implicature $F +> Q$ is explained if:

$$H(F) \Rightarrow Q. \tag{3.41}$$

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Section 4

Appendix

Rationalisability

Rationalisability

1. Rationalisability is an equilibrium concept weaker than Nash equilibrium.
2. Rationalisability characterises strategy pairs (S, H) for which:
   (i) each player choice of strategy is **rational given his beliefs** about the other players strategy and the state of the world.
   (ii) it is **common knowledge** that (i) holds.
3. Rationality means that each strategy is a **best response** to the assumed strategy of the other interlocutor.

Rationalisability Definition

1. Let $\mathscr{S}_0$ and $\mathscr{H}_0$ be the sets of all mixed speaker and hearer strategies.
2. Recursion step:
   - $\mathscr{S}_{n+1} = \{S \in \mathscr{S}_n \mid \exists H \in \mathscr{H}_n \; S \in \mathscr{B}_s(H)\}$.
   - $\mathscr{H}_{n+1} = \{H \in \mathscr{H}_n \mid \exists S \in \mathscr{S}_n \; H \in \mathscr{B}_H(S)\}$.
3. The set of rationalisable mixed strategies are:

$$\mathrm{Rat}_S = \bigcap_{i \in \mathbb{N}} \mathscr{S}_i \qquad \mathrm{Rat}_H = \bigcap_{i \in \mathbb{N}} \mathscr{H}_i \tag{4.42}$$

Canonical Rationalisabile Strategies

For a finite set X let $\overline{X}$ be the set of all probability distributions which assign equal probability to the elements of X.

1. Let $\mathscr{S}_0$ and $\mathscr{H}_0$ be the sets of all mixed speaker and hearer strategies.
2. Recursion step:
   - $\mathscr{S}_{n+1} = \{S \in \mathscr{S}_n \mid \exists H \in \mathscr{H}_n \, \forall \theta_S \colon S(\,.\,|\theta_S) \in \overline{\mathscr{B}_S(H, \theta_S)}\}$.
   - $\mathscr{H}_{n+1} = \{H \in \mathscr{H}_n \mid \exists S \in \mathscr{S}_n \, \forall \theta_H, F \colon H(\,.\,|\theta_H) \in \overline{\mathscr{B}_H(S, F, \theta_H)}\}$.
3. The set of rationalisable canonical mixed strategies are:

$$\overline{\mathrm{Rat}}_S = \bigcap_{i \in \mathbb{N}} \mathscr{S}_i \qquad \overline{\mathrm{Rat}}_H = \bigcap_{i \in \mathbb{N}} \mathscr{H}_i \tag{4.43}$$