Game Theoretic Pragmatics

Day 3

The Optimal Answer Model

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Course Outline

1. Introduction and Motivation
2. Basic Game Theory and the Iterated Best Response Model
3. The Basic Optimal Answer Model
4. Aspects of Bounded Rationality
5. Some Extensions of the Optimal Answer Model

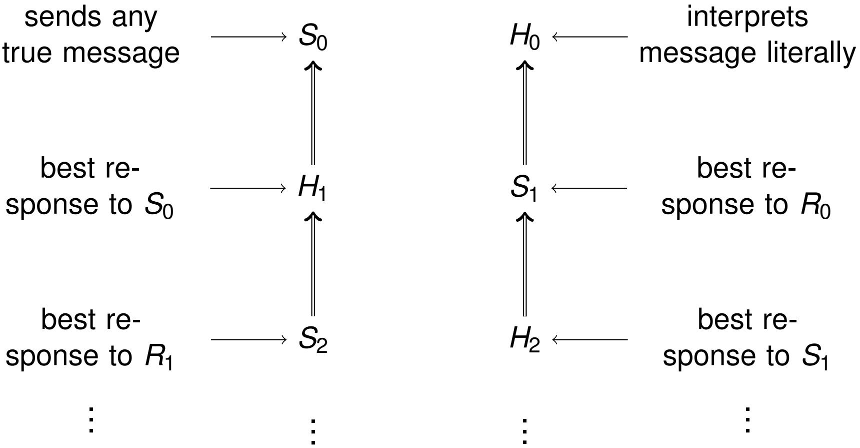
Iterated Best Response Model:

1. Start with a signalling game $\mathcal{G}$ in which the hearer makes his choice on the basis of literal meaning (H^0).
2. Calculate the speaker's best response S^1 to H^0.
3. Calculate the hearer's best response H^2 to S^1.
4. Iterate this process until it stabilises in a strategy pair $(S, H) = (S^n, H^{n\pm1})$
5. Implicature $F +\!> Q$ is explained if:

$$H(F) \Rightarrow Q. \tag{0.1}$$

The Iterated Best Response (IBR) Model

[Franke, 2009, p. 57]



Optimal Answer Approach

1. Start with a signalling game $\mathcal{G}$ in which the hearer makes his choice on the basis of literal meaning.
2. Impose pragmatic constraints and calculate optimal speaker strategy S by backward induction.
3. Implicature $F +> Q$ is explained if for all possible speaker strategies S which satisfy backward induction:

$$S^{-1}(F) \Rightarrow Q. \tag{0.2}$$

Models of Signalling Behaviour

GT Models

- non–evolutionary
  - online
    - immediate effects
    - two interlocutors
  - competence
    - several round of interaction
    - two interlocutors
- evolutionary
  - replication involving several generations
  - large populations (infinite)

Models of Signalling Behaviour

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Outline

1. Introduction & Repetition
2. The Optimal Answer Model
3. Implicatures
4. Applications
5. Normal Optimal Answer Models
6. Constructing a Normal OA Model

Section 1

Introduction & Repetition

[Benz and van Rooij, 2007]

Out-of-Petrol Example

Modified version

Example 1

A stands in front of his obviously immobilised car:

A: I am out of petrol.

B: There is a garage to the left round the corner. (G_l)

Possible alternative:

- There is a garage to the right round the corner. (G_r)

Properties:

- Garage can be **open** and **closed**.

Some Elements of the Model

World	Garage left	Garage right	Action g_l	Action g_r	random search r
w_1	open	open	1	1	ε
w_2	open	closed	1	0	ε
w_3	open	—	1	0	ε
w_4	closed	open	0	1	ε
w_5	closed	closed	0	0	ε
w_6	closed	—	0	0	ε
w_7	—	open	0	1	ε
w_8	—	closed	0	0	ε
w_9	—	—	0	0	ε

Meaning:

- open: at this place there is a garage and it is open.
- closed: at this place there is a garage and it is closed.
- — : at this place there is no garage.

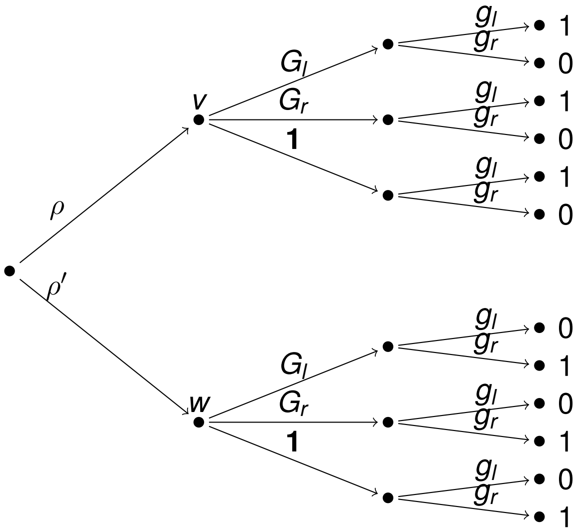
Some Simplifications for the Graphical Solution

- We consider again the graphical solution.
- We simplify the trees by considering the following worlds only:

World	Garage left	Garage right	Probab.	Action g_l	Action g_r	random search r
$v = w_2$	open	closed	ρ	1	0	ε
$w = w_4$	closed	open	ρ'	0	1	ε

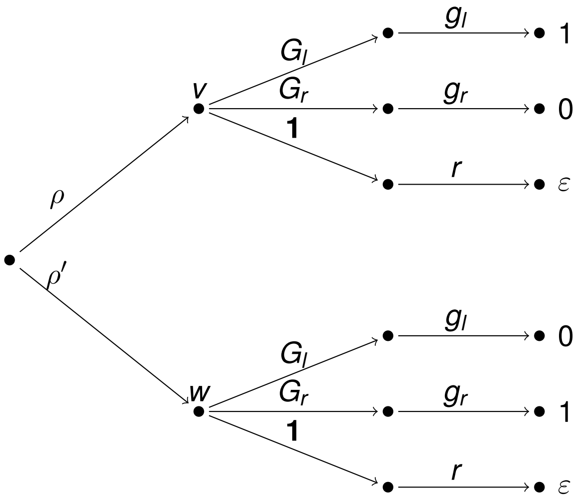
In the first tree, we simplify further by omitting random search r.

The Game Tree



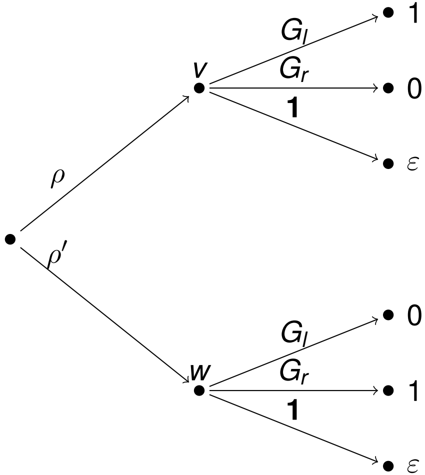
The Game Tree

Hearer chooses optimal act.



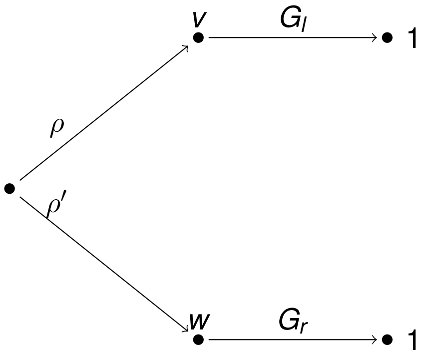
The Game Tree

Speaker calculating backwards



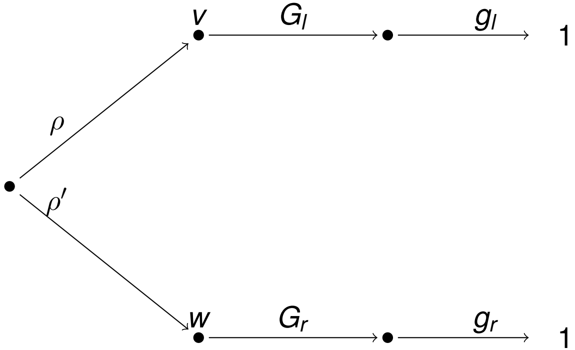
The Game Tree

Speaker choosing optimal action



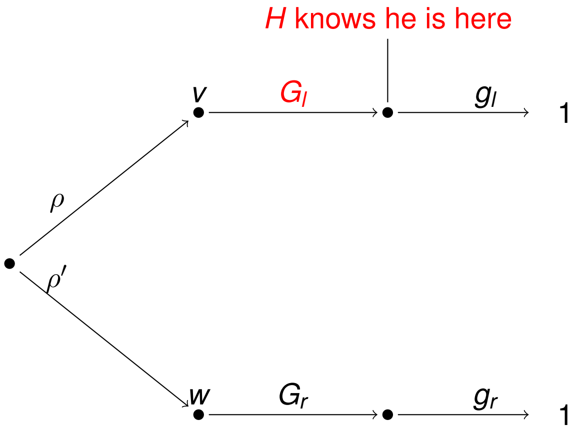
The Game Tree

Predicted behaviour



The Game Tree

Hearer can infer that the speaker is in v when uttering G_l!



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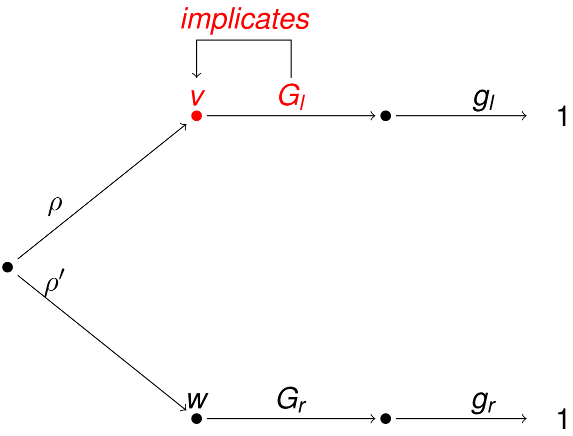


Assumptions

Actions, worlds, probabilities, and utilities:

World	Garage left	Garage right	Probability	Action g_l	Action g_r	random search r
v	open	closed	0.5	1	0	ε
w	closed	open	0.5	0	1	ε

Additional assumptions:

- $EU(r|G_l), EU(r|G_r) = EU(r) = \varepsilon \geq 0.5$.
- $EU(g_l|G_l), EU(g_r|G_r) > \varepsilon$.
- $EU(g_r|G_l) = EU(g_r)$ and $EU(g_l|G_r) = EU(g)$.

1. First, we calculated the optimal answers for the speaker.
2. The fact that an answer is optimal allows for inferences about the state of the speaker.
3. These inferences explain the implicature.
4. The optimal answers were found by Backward Induction.

$\Rightarrow$ The generalisation of this procedure leads to the Optimal Answer Model.

Section 2

The Optimal Answer Model

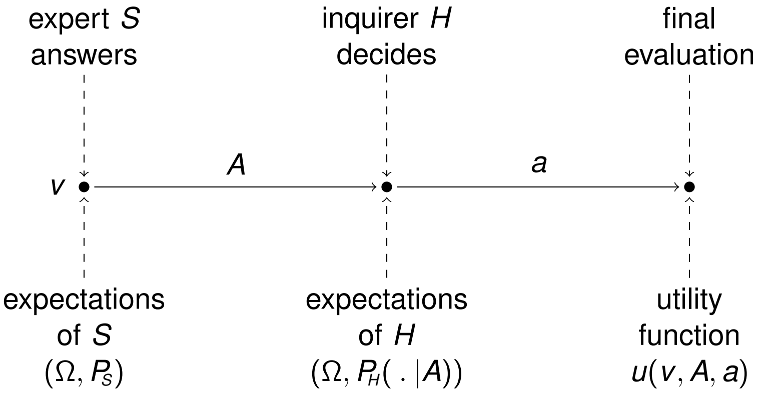
[Benz, 2006]

General Situation

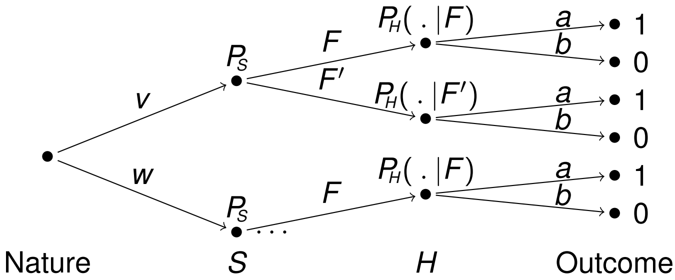
We consider situations in which:

- A person H, called inquirer, has to solve a decision problem $\langle(\Omega, P), \mathcal{A}, u\rangle$.
- A person S, called expert, provides H with information that helps solving S's decision problem.
- P_S represents S's expectations about Ω at the time when S answers.

General Situation

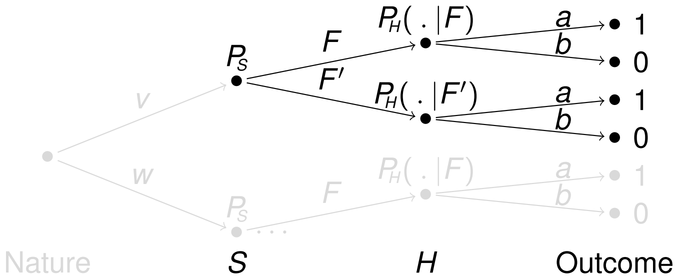


Relation to Game Tree



Relation to Game Tree

General situation consider in Optimal Answer Model:



General Characteristics

1. The answering situation poses two sequential decision problems:
   1. The speaker's problem of providing optimal answer.
   2. The hearer's problem of finding optimal action.
2. The decision problems are represented by:
   1. The speaker's problem: support problem.
   2. The hearer's problem: decision problem.
3. The most simple strategy for solving sequential decision problems is backward induction.
4. Solution defines set of optimal answers.
5. From optimal answerhood implicatures can be calculated.

Decision Problems

Definition 2

A decision problem is a triple $\langle(\Omega, P), \mathcal{A}, u\rangle$ such that:

- (Ω, P) is a finite probability space,
- $\mathcal{A}$ a finite, non–empty set, and
- $u : \mathcal{A} \times \Omega \longrightarrow \mathbb{R}$ a function.

$\mathcal{A}$ is called the action set, and its elements actions. u is called a payoff or utility function.

Support Problems

Definition 3 (Support Problem)

$\sigma = \langle \Omega, P_S, P_H, \mathcal{A}, u \rangle$ is a support problem if

- (Ω, P_S) is a finite probability space, and
- $\langle (\Omega, P_H), \mathcal{A}, u \rangle$ a decision problem.

We further assume:

$$\forall X \subseteq \Omega \; P_S(X) = P_H(X|K) \text{ for } K = \{v \in \Omega \mid P_S(v) > 0\}. \tag{2.3}$$

Calculation of Optimal Answers

The Inquirer's Decision Situation

Optimising expected utilities of actions

The expected utility of an action a is defined by:

$$EU(a) = \sum_{v \in \Omega} P(v) \times u(a, v). \tag{2.4}$$

After learning A, the inquirer optimises the conditional expected utility:

$$EU_H(a|A) = \sum_{v \in \Omega} P_H(v|A) \times u(a, v). \tag{2.5}$$

Hence, he will choose his actions from the set:

$$\mathscr{B}(A) := \{a \in \mathcal{A} \mid \forall b \in \mathcal{A}\ EU_H(b|A) \leq EU_H(a|A)\}. \tag{2.6}$$

The Expert's Decision Situation

Optimising expected utilities of answers

If there exists for each answer A a unique optimal choice $a_A \in \mathscr{B}(A)$, then the expected utility of an answer is defined as:

$$EU_s(A) := \sum_{v \in \Omega} P_s(v) \times u(v, a_A) = EU_s(a_A). \tag{2.7}$$

If the inquier's choice is not unique, then let $h(\,.\,|A)$ be a probability distribution over $\mathscr{B}(A)$ representing the inquirer's choice:

$$h(a|A) > 0 \Rightarrow a \in \mathscr{B}(A). \tag{2.8}$$

The expert has to optimise:

$$EU_s(A) := \sum_{a \in \mathscr{B}(A)} h(a|A) \times EU_s(a). \tag{2.9}$$

The Set of Optimal Answers

with representation of Gricean maxims

Maxim of Quality: Be truthful!

This restricts the expert's answers to:

$$Adm_\sigma := \{A \subseteq \Omega \mid P_S(A) = 1\} \tag{2.10}$$

Hence, the set of optimal answers is provided by:

$$\mathrm{Op}_\sigma := \{A \in \mathrm{Adm}_\sigma \mid \forall B \in \mathrm{Adm}_\sigma\, EU_S(B) \leq EU_S(A)\}. \tag{2.11}$$

Gricean Maxims

their replacement

The Gricean Maxims of

- (Quality)
- (Quantity)
- (Relevance)

have been replaced by

- (Quality): Restriction to true (admissible) answers.
- (Utility): Optimal answers are calculated by backward induction in support problems, which involves maximising expected utilities.

Optimal Answers in the Italian Newspaper Examples

Italian Newspaper

Mention–some

Example 4

Somewhere in the streets of Amsterdam...

- *H*: Where can I buy an Italian newspaper?
- *S*: At the station and at the Palace but nowhere else. (*SE*)
- *S*: At the station. (*A*) / At the Palace. (*B*)

The mention-some answers *A* and *B* are as good as the strongly exhaustive answer *SE*.

The Model

- *H*'s actions
  - *a*: going to the station,
  - *b*: going to the Palace.
- Possible worlds and utilities

Ω	Station	Palace	$u(v, a)$	$u(v, b)$
w_1	$+$	$+$	1	1
w_2	$+$	$-$	1	0
w_3	$-$	$+$	0	1
w_4	$-$	$-$	0	0

- Answers:
  - *A*: At the station ($A = \{w_1, w_2\}$),
  - *B*: At the Palace ($B = \{w_1, w_3\}$),

Mention–some as good as Strongly Exhaustive

Case: answer $A =$ At the station

We consider the general case in which there may be more actions and worlds than shown on the previous slide.

1. As $\forall x\ EU_H(x|A) \leq 1$ and $EU_H(a|A) = 1$, it follows $a \in \mathscr{B}(A)$.

Case 1: The hearer's expected utility of all other acts than a have a lower expected utility; i.e. $\mathscr{B}(A) = \{a\}$:

2. If $\mathscr{B}(A) = \{a\}$, then

$$EU_S(A) = \sum_{v \in \Omega} P_S(v) \times u(v, a) = \sum_{v \in A} P_S(v) \times u(v, a) = 1.$$

This shows that answer A is optimal if action a is the only optimal choice of the hearer.

Mention–some as good as Strongly Exhaustive

Case: answer A = At the station

Case 2: There are other actions c for which the hearer's expectations are equal to the hearer's expected utility of a.

3. Assume there exists $c \in \mathscr{B}(A)$ such that $c \neq a$. Then, this implies that for all v: $P_H(v|A) > 0 \Rightarrow u(v,c) = u(v,a) = 1$.
4. By (2.3) it holds: $P_H(v|A) > 0 \Rightarrow P_S(v) > 0$.
5. Hence,

$$EU_S(A) = \sum_{c \in \mathscr{B}(A)} h(c|A) \sum_{v \in \Omega} P_S(v) \times u(v,c)$$

$$= \sum_{c \in \mathscr{B}(A)} h(c|A) \sum_{v \in A} P_S(v) \times \underbrace{u(v,a)}_{=1} = 1.$$

Clearly, no other answer can yield a higher payoff.

Mention–some as good as Strongly Exhaustive

Case: answer A = At the station

Result:

1. We have seen that A is an optimal answer in both cases.
2. The proof that answers B and SE are optimal if the speaker knows that B or SE is analogous.
3. In particular: if the speaker knows that SE, then all three answers are optimal!

Section 3

Implicatures

[Benz and van Rooij, 2007, Benz, 2009]

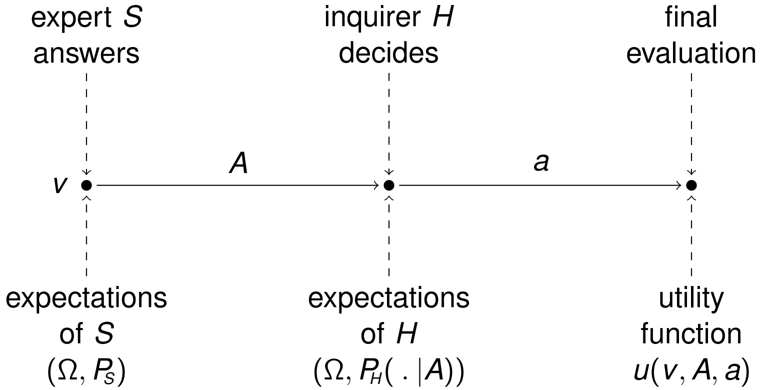
Implicatures

[Grice, 1989, p. 86]

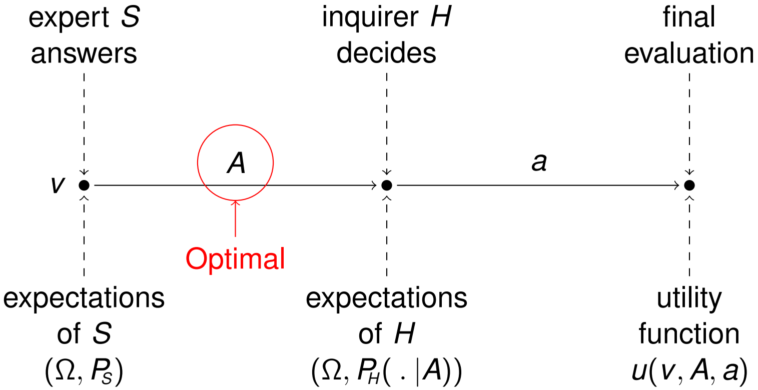
What is an implicature?

"... what is implicated is what is required that one assume a speaker to think in order to preserve the assumption that he is observing the Cooperative Principle (and perhaps some conversational maxims as well), ..."

General Situation



General Situation



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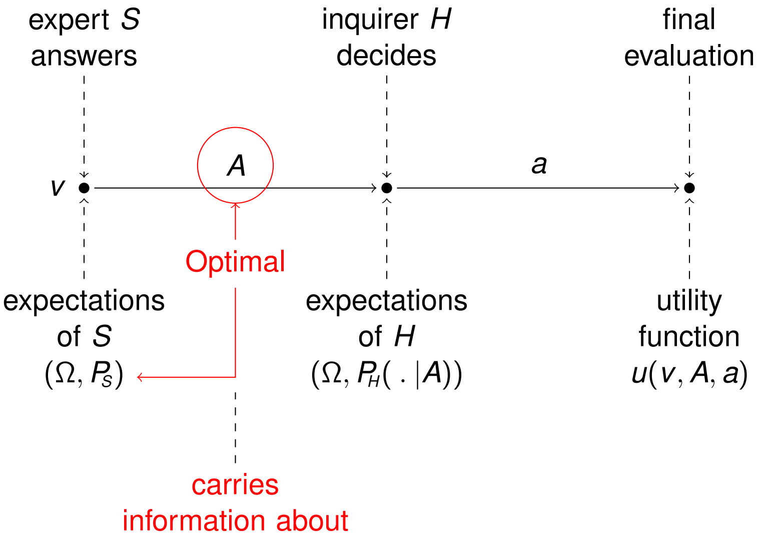

Definition of Implicatures

Definition 5 (Implicature)

Let $\mathcal{S}$ be a given set of support problems with joint decision problem $\langle(\Omega, P_H), \mathcal{A}, u\rangle$. Let $A, R \subseteq \Omega$ be two propositions with $A \in \mathrm{Op}_\sigma$ for some $\sigma \in \mathcal{S}$. Then we set:

$$A +> R \Leftrightarrow \forall\sigma \in \mathcal{S}\,(A \in \mathrm{Op}_\sigma \rightarrow P_S^\sigma(R) = 1), \tag{3.12}$$

- Op_σ: Set of optimal answers in situation σ.

- Now interested in a simple criterion for calculating implicatures in case speakers is expert.

$\Rightarrow$ Often assumed that speaker is expert.

$\Rightarrow$ Leads to much stronger implicatures.

Expert Assumption

- $O(a)$: the set of all worlds in which a is an optimal action:

$$O(a) = \{v \in \Omega \mid \forall b \in \mathcal{A}\, u(v, b) \leq u(v, a)\}. \qquad (3.13)$$

Definition 6 (Expert)

Let $\mathcal{S}$ be a set of support problems with joint decision problem $\langle(\Omega, P_H), \mathcal{A}, u\rangle$. Then we call S an *expert* in a support problem $\sigma \in \mathcal{S}$ if

$$\exists a \in \mathcal{A}\; P_S^{\sigma}(O(a)) = 1.$$

Some Useful Sets

- $\mathscr{B}(A)$: the set of best choices for the hearer after learning A.

Then let:

$$A^* := \{v \in A \mid P_H(v) > 0\} \text{ and } A^+ = \bigcap \{O(a) \mid a \in \mathscr{B}(A)\}, \quad (3.14)$$

- A^*: the common ground updated with A.
- A^+: information that follows from recommending actions in $\mathscr{B}(A)$.

Special Case

Expert knows optimal Action

Lemma 7

Let $\mathcal{S}$ be a set of support problems with joint decision problem $\langle(\Omega, P_H), \mathcal{A}, u\rangle$. Assume furthermore that

1. *S is an expert for every $\sigma \in \mathcal{S}$,*
2. $\forall v \in \Omega\, \exists \sigma \in \mathcal{S}\, P_S^\sigma(v) = 1$.

Let $\sigma \in \mathcal{S}$ and $A, R \subseteq \Omega$ be two propositions with $A \in \mathrm{Op}_\sigma$. Then, it follows that:

$$A +> R \text{ iff } A^* \cap A^+ \subseteq R.$$

Section 4

Applications

We concentrate on the special cases in which the preconditions of the special lemma are met:

- the speaker is an expert, i.e. $\exists a \in \mathcal{A}\ P_S^\sigma(O(a)) = 1$;
- $\forall v \in \Omega\ \exists \sigma \in \mathcal{S}\ P_S^\sigma(v) = 1$.

The Out–of–Petrol Example

Example 8

A: I am out of petrol.
B: There is a garage round the corner. (G)
$+>$ The garage is open. (R)

Ω	G	R	go-to-g	search
w_1	$+$	$+$	1	ε
w_2	$+$	$-$	0	ε
w_3	$-$	$-$	0	ε

Assumptions:

- $EU_H(\text{go-to-g}|G) > \varepsilon > EU_H(\text{go-to-g})$.
- $EU_H(\text{search}|G) = EU_H(\text{search}) = \varepsilon$ with $1 > \varepsilon > 0$.

Reasoning

1. $G^* = \{w_1, w_2\}$ (garage round corner)
2. $\mathscr{B}(G) = \{\text{go-to-g}\}$. (set of optimal actions)
3. $O(\text{go-to-g}) = \{w_1\} = R$.
   (set of worlds in which go-to-g is optimal)
4. $\Rightarrow G^+ = \bigcap\{O(a) \mid a \in \mathscr{B}(G)\} = \{w_1\}$.
5. $\Rightarrow G^* \cap G^+ = \{w_1\} = R$.
6. $\Rightarrow G +> R$. (Lem. 7)
7. Hence $P_S^\sigma(R) = 1$ (speaker believes R)

Italian Newspaper Example

Example 9

Somewhere in the streets of Amsterdam...

1. I: Where can I buy an Italian newspaper?
2. E: At the station and at the Palace but nowhere else. (S)
3. E: At the station. ($\mathrm{IN}(s)$) / At the Palace. ($\mathrm{IN}(p)$)

Ω	$\mathrm{IN}(p)$	$\mathrm{IN}(s)$	go-to-p	go-to-s	search
w_1	$+$	$+$	1	1	ε
w_2	$+$	$-$	1	0	ε
w_3	$-$	$+$	0	1	ε
w_4	$-$	$-$	0	0	ε

Assumption: The probabilities of all four worlds are equal, and $\frac{1}{2} < \varepsilon < 1$.

Reasoning

1. For $d = p, s$, it is $EU_H(\text{go-to-}d|\mathrm{IN}(d)) = 1 > \varepsilon$ and $\mathscr{B}(\mathrm{IN}(d)) = \{\text{go-to-}d\}$.
2. We find $O(\text{go-to-}p) = \{w_1, w_2\} = [\![\mathrm{IN}(p)]\!]$ and $O(\text{go-to-}s) = \{w_1, w_3\} = [\![\mathrm{IN}(s)]\!]$.
3. As $[\![\mathrm{IN}(p)]\!] \not\subseteq [\![\neg\mathrm{IN}(s)]\!]$ and $[\![\mathrm{IN}(s)]\!] \not\subseteq [\![\neg\mathrm{IN}(p)]\!]$,
4. it follows by Lemma 7 that neither $\mathrm{IN}(p)$ implicates $\neg\mathrm{IN}(s)$, nor $\mathrm{IN}(s)$ implicates $\neg\mathrm{IN}(p)$.

Hip Hop & Beijing Opera

Example 10

1. A: I want to see a classical Beijing opera tonight or Chinese acrobatics, but I don't want to go to one of these modern tea houses which mix both things. What can I do tonight?
   B: You can go to the Lantern Tea House!
2. John loves to dance to Salsa music and he loves to dance to Hip Hop, but he can't stand it if a club mixes both styles.
   J: I want to dance tonight. Is the Music in Roter Salon ok?
   E: Tonight they play Hip Hop at the Roter Salon.

Hip Hop

$H(d)$: There is Hip Hop at d; $S(d)$: There is Salsa at d.

Ω	$H(r)$	$S(r)$	*stay-home*	$Good(r)$	$H(r) \in \mathrm{Op}_{w_j}$
w_1	1	1	ε	0	no
w_2	1	0	ε	1	yes
w_3	0	1	ε	1	no
w_4	0	0	ε	0	no

Hip Hop

Notation:

$$Good(d) \text{ iff } (H(d) \vee S(d)) \wedge \neg(H(d) \wedge S(d)).$$

Assumptions:

- $\forall d \neq d' : \; P_H(Good(d)|H(d')) = P_H(Good(d)) < P_H(Good(d')|H(d')).$
- $\forall d : \; P_H(Good(d)|H(d)) > \varepsilon > P_H(Good(d)).$

Reasoning

1. As $EU_H(r|H(r)) = P_H(r|H(r))$, it follows that $\mathscr{B}(H(r)) = \{r\}$.
2. $O(r) = \{w_2\} =: R$.
3. With Lem 7 it follows that $H(r) +> R$, hence $H(r) +> Good(r)$.

Section 5

Normal Optimal Answer Models

[Benz, 2009]

Rules for Setting up Game Theoretic Models

The Problem

How to get from the examples to the models?

Rules for Setting up Game Theoretic Models

Questions:

1. Are there fixed rules for setting up game theoretic models?
2. Are there hidden default assumptions about game theoretic parameters when interpreting discourse and generating its implicatures?

The Out–of–Petrol–Example

Example 11 (Out-of-Petrol)

H: I am out of petrol.
S: There is a garage round the corner. (*G*)
$+>$ The garage is open. (*H*)

Which assumptions have to be made to arrive at this model:

Ω	$G(d)$	$H(d)$	go-to-d	search
w_1	$+$	$+$	1	ε
w_2	$+$	$-$	0	ε
w_3	$-$	$+$	0	ε
w_4	$-$	$-$	0	ε

Directed Actions

1. Actions: the hearer's action set is often such that they are directed towards an object:
   - Out–of–Petrol: go to places where to look for petrol.
   - Italian newspaper: go to places where to look for Italian newspapers.
   - Hip–Hop: go to places for dancing in the evening.
2. Context or utterances provide information about which objects are desired and which are not.

Representation

1. Objects of directed actions: given by a set D.
2. Actions: contain all actions $act\text{-}d$ for $d \in D$.
   - Out–of–Petrol: *going-to-d*, d a possible location of a petrol station.
   - Italian newspaper: *going-to-d*, d a newspaper shop.
   - Hip–Hop: *going-to-d*, d a dance location in town.
3. Context or utterances provide information about predicate *Good* for which:

$$act\text{-}d \text{ is good } \textit{ iff } Good(d). \tag{5.15}$$

Objectives

1. Good: Objects are good because they have certain properties:
   - Out–of–Petrol: d is a good place because there is a garage at d ($G(d)$) which is open ($Opn(d)$).
   - Italian newspaper: d is a good newspaper shop because it sells Italian newspapers ($\mathrm{IN}(d)$).
   - Hip–Hop: d is a good dance location because there plays either Hip Hop ($H(d)$) or Salsa ($S(d)$) but not both.
2. The *Good* predicate is defined by these properties:
   - Out–of–Petrol: $Good(d)$ *iff* $G(d) \wedge Opn(d)$.
   - Italian newspaper: $Good(d)$ *iff* $\mathrm{IN}(d)$.
   - Hip–Hop: $Good(d)$ *iff* $(H(d) \vee S(d)) \wedge \neg(H(d) \wedge S(d))$.

Utilities

What does the *Good* predicate have to do with utilities?

$$u(v, act\text{-}d) = \begin{cases} 1 & \text{if } v \models Good(d) \\ 0 & \text{if } v \models \neg Good(d) \end{cases} \tag{5.16}$$

This leads to the next question: What are the possible worlds Ω?

Possible Worlds

- Success depends on *Good* predicate.
- The *Good* predicate is defined in terms of elementary properties $A_i(d)$ of domain objects $d \in D$.
- If $A_1, \ldots, A_n$ are the elementary properties, and $d_1, \ldots, d_m$ the direct objects of the hearer's actions, then the possible worlds are defined by:

Ω	$A_1(d_1)$	$\ldots$	$A_n(d_1)$	$A_1(d_2)$	$\ldots$	$A_n(d_2)$	$\ldots$	$A_n(d_m)$
v_1	1	$\ldots$	1	1	$\ldots$	1	$\ldots$	1
v_2	1	$\ldots$	1	1	$\ldots$	1	$\ldots$	0
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	
$v_{2^{n\,m}}$	0	$\ldots$	0	0	$\ldots$	0	$\ldots$	0

Probabilities

What are the probabilities of the possibile worlds?

- Nothing is known about the probabilities of the elementary events $A_i(d_j)$.
- Assume that they all are equally probable:

$$\exists \alpha > 0 \, \forall d_j \in D \, \forall i : \; P_H(A_i(d_j)) = \alpha. \tag{5.17}$$

- Assume further that the elementary events are probabilistically independent:

$E(n, D) := \{A_i(d) \mid d \in D, 1 \leq i \leq n\}$ the set of elementary events.

$$\forall \mathcal{E} \subseteq E(n, D) : \; P_H\left(\bigcap \mathcal{E}\right) = \prod_{X \in \mathcal{E}} P_H(X). \tag{5.18}$$

The Neutral Alternative

1. In all our previous models, the set of hearer's actions contained a neutral alternative **n**:
   - Out–of–Petrol: doing a random search in town.
   - Italian newspaper: choose newspaper shop at random.
   - Hip–Hop: staying at home instead of going out.
2. The payoff ε of this act was between the expected payoff of the other actions and the highest possible payoff 1.

Problem: How to determine the payoff of the neutral act?

The Neutral Alternative

General Idea:

1. The neutral act **n** is for no world the best act; i.e.

$$\forall v \in \Omega\, \exists a \in \mathcal{A} : \; u(v, \mathbf{n}) < u(v, a). \tag{5.19}$$

2. The neutral act **n** must be a possible choice for the hearer in the situation in which he was before learning any new information. This means:

$$EU_H(a) \leq EU_H(\mathbf{n}) \;\text{ for all }\; a \in \mathcal{A}. \tag{5.20}$$

3. The speaker may believe either of two things:
   1. The neutral act **n** is the best choice for the hearer; i.e.

   $$EU_s(a) < EU_s(\mathbf{n}) \;\text{ for all }\; a \in \mathcal{A} \setminus \{\mathbf{n}\}. \tag{5.21}$$

   2. There are better choices than the neutral act **n**; i.e.

   $$\exists a \in \mathcal{A} \setminus \{\mathbf{n}\} : \; EU_s(a) > EU_s(\mathbf{n}) \tag{5.22}$$

Summary of Important Implicit Assumptions

I. *Equality and completeness of objectives:*
   a) Objectives have equal weight.
   b) Objectives are complete: There are no other objectives that influence the decision.

II. *Laplacian Assumptions:*
   a) Equal probability of elementary events.
   b) Probabilistic independence of elementary events.

III. *Neutral alternative action:*
   a) Before *H* learns anything, the neutral act is among his optimal choices.
   b) *S* either believes the neutral act to be the only optimal act, or he believes that it is not optimal.

Section 6

Constructing a Normal OA Model

The Beijing Opera Example

Beijing Opera

Example 12

A: I want to see a classical Beijing opera tonight or Chinese acrobatics, but I don't want to go to one of these modern tea houses which mix both things. What can I do tonight?
B: You can go to the Lantern Tea House!

Hearer's Action Set

Example 13

A: I want to see a classical Beijing opera tonight or Chinese acrobatics, but I don't want to go to one of these modern tea houses which mix both things. What can I do tonight?
B: You can go to the Lantern Tea House!

Hearer's action set:

⇒ Action set $\mathcal{A}$: *going-to-d* for tea houses *d*.

Elementary Properties

Example 14

A: I want to see a classical Beijing opera tonight or Chinese acrobatics, but I don't want to go to one of these modern tea houses which mix both things. What can I do tonight?
B: You can go to the Lantern Tea House!

Elementary properties:

⇒ $B(d)$: d shows classical Beijing opera;
$C(d)$: d shows Chinese acrobatics.

Utilities

Example 15

A: I want to see a classical Beijing opera tonight or Chinese acrobatics, but I don't want to go to one of these modern tea houses which mix both things. What can I do tonight?
B: You can go to the Lantern Tea House!

Elementary properties:

$B(d)$: d shows classical Beijing opera;
$C(d)$: d shows Chinese acrobatics.

Good predicate:

$\Rightarrow$ $Good(d)$ *iff* $(B(d) \vee C(d)) \wedge \neg(B(d) \wedge C(d))$.

Domain Objects

Example 16

A: I want to see a classical Beijing opera tonight or Chinese acrobatics, but I don't want to go to one of these modern tea houses which mix both things. What can I do tonight?
B: You can go to the Lantern Tea House!

Direct domain objects:

$\Rightarrow$ $D = \{\text{Lantern Tea House}\} =: \{d\}$.

Possible Worlds

Elementary properties:

$B(d)$: d shows classical Beijing opera;
$C(d)$: d shows Chinese acrobatics.

Direct domain objects:

$\Rightarrow$ $D = \{\text{Lantern Tea House}\} =: \{d\}$.

Possible Worlds:

Ω	$B(d)$	$C(d)$
w_1	1	1
w_2	1	0
w_3	0	1
w_4	0	0

Probabilities

Probabilities:

1. No probabilities stated in the example.
2. From list of implicit assumptions:

$$\exists\alpha > 0\, \forall d \in D\, \forall A :\; P_H(A(d)) = \alpha. \tag{6.23}$$

3. If nothing else is known, assume $\alpha = 1/2$.
4. Identify probabilities with hearer's probabilities P_H

Probabilities:

Ω	$B(d)$	$C(d)$	P_H	$\alpha = 1/2$
w_1	1	1	α^2	$1/4$
w_2	1	0	$\alpha\,(1-\alpha)$	$1/4$
w_3	0	1	$(1-\alpha)\,\alpha$	$1/4$
w_4	0	0	$(1-\alpha)^2$	$1/4$

Summing Up

Elementary properties:

$B(d)$: d shows classical Beijing opera;
$C(d)$: d shows Chinese acrobatics.

Direct domain objects:

$\Rightarrow$ $D = \{\text{Lantern Tea House}\} =: \{d\}$.

The Model:

Ω	$B(d)$	$C(d)$	$Good(d)$	**n**	P_H
w_1	1	1	0	ε	$1/4$
w_2	1	0	1	ε	$1/4$
w_3	0	1	1	ε	$1/4$
w_4	0	0	0	ε	$1/4$

The Utility of the Neutral Act

- All properties of the model are determined;
- Exception: Utility of the neutral act!

From

$$\forall v \in \Omega\, \exists a \in \mathcal{A} : \ u(v, \mathbf{n}) < u(v, a). \tag{6.24}$$

and

$$EU_H(a) \leq EU_H(\mathbf{n}) \ \text{ for all } \ a \in \mathcal{A}. \tag{6.25}$$

it follows:

$$1/4 \leq \varepsilon < 1. \tag{6.26}$$

The Utility of the Neutral Act

We have

$$1/4 \leq \varepsilon < 1. \tag{6.27}$$

Finally, we have to satisfy the speaker constraints for the neutral act, i.e. the disjunction of the following two conditions:

1. $EU_s(a) < EU_s(\mathbf{n})$ for all $a \in \mathcal{A} \setminus \{\mathbf{n}\}$;
2. $\exists a \in \mathcal{A} \setminus \{\mathbf{n}\} : EU_s(a) > EU_s(\mathbf{n})$.

This implies:

$$1/4 < \varepsilon < 1. \tag{6.28}$$

This completes the construction of the model!

The Model

Elementary properties:

$B(d)$: d shows classical Beijing opera;
$C(d)$: d shows Chinese acrobatics.

The Model with $1/4 < \varepsilon < 1$:

Ω	$B(d)$	$C(d)$	$Good(d)$	**n**	P_H
w_1	1	1	0	ε	$1/4$
w_2	1	0	1	ε	$1/4$
w_3	0	1	1	ε	$1/4$
w_4	0	0	0	ε	$1/4$

All parameters are set except the exact value of ε

Generalisation from Example

A complete list of all *Good* predicates and their optimal answers in the Out-of-Petrol type of examples.

Good				*A*				*B*				$\neg A$				$\neg B$			
1	1	1	1	✓	✓	·	·	✓	·	✓	·	·	·	✓	✓	·	✓	·	✓
1	1	1	0	✓	✓	·	·	✓	·	✓	·	·	·	—	✓	·	—	·	✓
1	1	0	1	✓	✓	·	·	—	·	✓	·	·	·	✓	—	·	✓	·	✓
1	1	0	0	✓	✓	·	·	—	·	✓	·	·	·	✓	✓	·	—	·	✓
1	0	1	1	—	✓	·	·	✓	·	✓	·	·	·	✓	✓	·	✓	·	—
1	0	1	0	—	✓	·	·	✓	·	✓	·	·	·	—	✓	·	✓	·	✓
1	0	0	1	—	✓	·	·	—	·	✓	·	·	·	✓	—	·	✓	·	—
1	0	0	0	✓	—	·	·	✓	·	—	·	·	·	✓	✓	·	✓	·	✓
0	1	1	1	✓	—	·	·	✓	·	—	·	·	·	✓	✓	·	✓	·	✓
0	1	1	0	✓	—	·	·	✓	·	—	·	·	·	—	✓	·	—	·	✓
0	1	0	1	✓	—	·	·	✓	·	✓	·	·	·	✓	—	·	✓	·	✓
0	1	0	0	—	✓	·	·	✓	·	✓	·	·	·	✓	✓	·	✓	·	—
0	0	1	1	✓	✓	·	·	✓	·	—	·	·	·	✓	✓	·	✓	·	—
0	0	1	0	✓	✓	·	·	—	·	✓	·	·	·	✓	—	·	✓	·	✓
0	0	0	1	✓	✓	·	·	✓	·	✓	·	·	·	—	✓	·	—	·	✓
0	0	0	0	✓	✓	·	·	✓	·	✓	·	·	·	✓	✓	·	✓	·	✓

Example 17

1. I: I want to have a house with either both a garden and a balcony, or with neither a garden nor a balcony.
2. I: I want to have a house with both a garden (A) and a balcony (B).
3. E: The house in Shakespeare Avenue has a balcony (A).
   1) $+>$ The house in Shakespeare Avenue does not have a garden.
   2) $+>$ The house in Shakespeare Avenue does have a garden.

But: In both cases, a clarification request seems to be most natural.

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