Game Theoretic Pragmatics

Day 5

Extensions of the Optimal Answer Model

Anton Benz

Centre for General Linguistics (ZAS), Berlin

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Course Outline

1. Introduction and Motivation
2. The Basic Iterated Best Response Model
3. The Basic Optimal Answer Model
4. Aspects of Bounded Rationality
5. Some Extensions of the Optimal Answer Model
Optimal Answer Approach

Explanation of Implicatures, [Benz and van Rooij, 2007]

Optimal Answer Approach

1. Start with a signalling game $G$ in which the hearer makes his choice on the basis of literal meaning.

2. Impose pragmatic constraints and calculate optimal speaker strategy $S$ by backward induction.

3. Implicature $F \rightarrow Q$ is explained if for all possible speaker strategies $S$ which satisfy backward induction:

$$S^{-1}(F) \Rightarrow Q. \quad (0.1)$$
The Optimal Answer Approach

expert $S$ answers

inquirer $H$ decides

final evaluation

Optimal

carries information about

expectations of $S$ $(\Omega, P_S)$

expectations of $H$ $(\Omega, P_H(\cdot|A))$

utility function $u(v, A, a)$
Today, we see several extensions to the basic OA Model:

1. Suspension of implicatures in a preferential non–monotonic model.
2. Clarification request and noisy signalling strategies.
Models of Signalling Behaviour

GT Models

- non–evolutionary
  - online
    - immediate effects
    - two interlocutors
  - competence
    - several round of interaction
    - two interlocutors
- evolutionary
  - replication involving several generations
  - large populations (infinite)
Models of Signalling Behaviour

GT Models

non–evolutionary

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Outline

1. Suspension and Nonmonotonicity
2. Efficient Clarification Requests and Expected Noise
3. Examples: Noisy Communication
Section 1

Suspension and Nonmonotonicity

[Benz, 2009]
Communicated Meaning

Grice distinguishes between:

- What is said.
- What is implicated.

Example 1

“Some of the boys came to the party.”

- said: at least two came
- implicated: not all came
1. Some of the boys came to the party.
   \( \rightarrow \) Not all of the boys came.

2. Some, perhaps all of the boys came to the party.
   \( \rightarrow \) It is possible that all came, and it is possible that not all came.
   \( \rightarrow \) ♦ all came & ♦ not all came.

3. I believe that some of the boys came to the party.
   \( \rightarrow \) ♦ all came & ♦ not all came.
A Hirschberg Style Example

Especially interested in:

1. A: Does this job candidate speak Spanish?
   1 He speaks Portuguese.
   $\Rightarrow$ He does not speak Spanish.
   2 B: I know he speaks Portuguese.
      $\Rightarrow$ B does not know whether he speaks Spanish.

2. A: How did the students do in the exam?
   1 B: Some students passed.
      $\Rightarrow$ Not many passed.
   2 B: I know that some students passed.
      $\Rightarrow$ B does not know whether many passed.
Suspension of Implicatures
Suspension and Cancellation

Example 2

“Some of the boys came to the party.”

1. Cancellation: Some, in fact all, of the boys came to the party.

2. Suspension: Some, perhaps all, of the boys came to the party.
Gazdar’s Incremental Account

Speaker has uttered $A$:

1. $e_0 := \{A\}$
2. $e_1$: Add all logical consequences to $e_0$.
3. $e_2$: Add all clausal implicatures which don’t contradict $e_1$.
4. $e_3$: Add all scalar implicatures which don’t contradict $e_2$.

Scalar implicatures are cancelled if they contradict logical consequences or clausal implicatures.
## Suspension and Clausal Implicatures

<table>
<thead>
<tr>
<th>a) stronger form</th>
<th>b) weaker form</th>
<th>c) implicature of weaker form</th>
</tr>
</thead>
<tbody>
<tr>
<td>know $A$</td>
<td>believe $A$</td>
<td>$\Diamond A \land \Diamond \neg A$</td>
</tr>
<tr>
<td>necessarily $A$</td>
<td>possibly $A$</td>
<td>$\Diamond A \land \Diamond \neg A$</td>
</tr>
<tr>
<td>$A$ and $B$</td>
<td>$A$ or $B$</td>
<td>$\Diamond A \land \Diamond \neg A \land \Diamond B \land \Diamond \neg B$</td>
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</tbody>
</table>

1. Some, possibly all of the boys came to the party.
   $\vdash \Diamond \text{ all came} \land \Diamond \text{ not all came}.$

2. I believe that some of the boys came to the party.
   $\vdash \Diamond \text{ all came} \land \Diamond \text{ not all came}.$
1. In more recent papers, the distinction is drawn between:
   i. **Weak** Implicatures: Some $\rightarrow \neg \square S \text{ All.}$
   ii. **Strong** Implicatures: Some $\rightarrow \square S \neg \text{ All.}$

2. **Rule**: If consistent with what is known, draw strong implicature!

$\Rightarrow$ Cannot account for differences in Hirschberg–Style examples!
The Hirschberg–Style Example
Extension to Relevance Implicatures

1. A: Does this job candidate speak Spanish?
   1  He speaks Portuguese.
      \[\rightarrow\] He does not speak Spanish.
   2  B: I know he speaks Portuguese.
      \[\rightarrow\] B does not know whether he speaks Spanish.

2. A: How did the students do in the exam?
   1  B: Some students passed.
      \[\rightarrow\] Not many passed.
   2  B: I know that some students passed.
      \[\rightarrow\] B does not know whether many passed.
Problem

- **Know** does not create clausal implicatures.
- **(Quality) ⇒** Answers are equivalent.
The Non–Monotonic Component

Normality
Example 3

“Some of the boys came to the party.”

1. $\Box A(\forall) \rightarrow \text{Utter}_S A(\forall)$ (Quantity)
2. $\text{Utter}_S A(\exists)$ (fact)
3. $\neg \Box A(\forall)$ (follows from lines 1 & 2)
4. $\Box A(\exists)$ (follows from 2 and Quality)
5. $\Box A(\neg \exists) \lor \Box A(\exists \land \neg \forall) \lor \Box A(\forall)$ (Expert)
6. $\Box A(\exists \land \neg \forall)$ (follows from lines 3., 4., and 5.)

**Expert**: Assumption that the speaker is an expert, i.e. knows the true state of the world.

Compare also [de Jager, 2007].
A Classical Explanation
Suspension of Scalar Implicatures

Example 4
“Some, perhaps all, of the boys came to the party.”

1. □A(∃) ∧ ◊A(∀) (logical form of utterance and Quality)
2. □A(¬∃) ∨ □A(∃ ∧ ¬∀) ∨ □A(∀) (Expert)
3. □A(∀) (follows from previous lines)
4. □A(∀) → Utters A(∀) (Quantity)
5. Contradiction (because speaker did not utter A(∀))
6. ¬(Expert) ≡ ◊¬A(¬∃) ∧ ◊¬A(∃ ∧ ¬∀) ∧ ◊¬A(∀)
7. □A(∃) ∧ ◊A(¬∀) ∧ ◊A(∀) (from the first and the previous line)
A Graphical Interpretation
Scalar Implicature: speaker says $A(\exists).$

\[ \bot \equiv \text{contradicts maxims} \]
Cancellation: speaker says $A(\exists) \land \square A(\forall)$.

$\bot \equiv$ contradicts maxims
A Graphical Interpretation

Suspension: speaker says $A(∃) \land \Diamond A(∀)$.

$\bot \equiv \text{contradicts maxims}$
Normality

Definition 5 (Preferential Models)
Let $S$ be the set of all support problems, then $\langle S, C, \sqsubseteq \rangle$ is a preferential model of support problems if

1. $C$ a partition of $S$,
2. $\sqsubseteq$ a well-founded linear order of $C$.

We set

$$\text{Min}(F) := \min\{ C \in C \mid \exists \sigma \in C \; F \in \text{Op}_\sigma \}$$

with:

- $\text{Op}_\sigma$: optimal answers defined as before by backward induction;
- $\text{Adm}^C_\sigma := \{ F \mid P^\sigma_S (F) = 1 \} \setminus \{ F \mid \exists C', \hat{\sigma} : \hat{\sigma} \in C' \sqsubseteq [\sigma]_C \land F \in \text{Op}_{\hat{\sigma}} \}$.
- $[\sigma]_C := C$ iff $\sigma \in C$. 

Normality

**Definition 6 (The Principle of Normality)**

Let \( \langle S, C, \sqsubseteq \rangle \) be a preferential model of support problems, \( F \in \mathcal{F} \), and \( \sigma \in \text{Min}(F) \), then an utterance of \( F \) implicates that \( H \) iff

\[
\forall \hat{\sigma} \in [\sigma] \cap \text{Min}(F) : A \in \text{Op}_{\hat{\sigma}} \rightarrow P_{E}(H) = 1, \tag{1.2}
\]

with \([\sigma]\) the set of all support problems that only differ in \( P_E \) from \( \sigma \).
The Job Interview Example

The Job Interview Example
Example 7

$H$: Does this job candidate speak Spanish?

   $\Rightarrow$ He does not speak Spanish.

2. $S$: I know he speaks Portuguese.
   $\Rightarrow$ $S$ does not know whether he speaks Spanish.

Assumptions

1. There are two job candidates $a, b$;
2. $S$ knows all about first candidate $a$;
3. $H$ knows all about second candidate $b$;
4. Question is about first candidate $a$. 
## A Model I

Case $S(a)$ true; only un-boxed forms; speaker expert about candidate $a$.

<table>
<thead>
<tr>
<th>$S(a)$</th>
<th>$P(a)$</th>
<th>$S(b)$</th>
<th>$P(b)$</th>
<th>$S(a)$</th>
<th>$P(a)$</th>
<th>$\neg S(a)$</th>
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1. All worlds equally probable
2. Spanish speaker much preferred over non-Spanish speaker.
3. Portuguese is a plus.
## A Model II

Case $S(a)$ false; only un-boxed forms; speaker expert about candidate $a$.

<table>
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<tr>
<th>$S(a)$</th>
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<th>$S(b)$</th>
<th>$P(b)$</th>
<th>$S(a)$</th>
<th>$P(a)$</th>
<th>$\neg S(a)$</th>
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Entries for modal sentences $\Box S(a), \Box P(a), \Box \neg S(a), \Box \neg P(a)$ for both tables identical to un-boxed forms.
Result

1. Predictions:
   1. $P(a)$ implicates that $\neg S(a)$;
   2. $\neg P(a)$ implicates that $S(a)$;
   3. $S(a)$ and $\neg S(a)$ do not lead to additional implicatures.

2. (Manner) implies modal forms will not be used.
   $\Rightarrow$ No implicatures defined for modal forms.
   $\Rightarrow$ Use of modal forms leads into contradiction.
   $\Rightarrow$ Solved by dropping normality (= expert) assumption.
   $\Rightarrow$ $\square P(a)$ and $\neg (\text{Expert})$ ($\equiv \Diamond \neg S(a) \vee \Diamond \neg P(a)$) implies $\Diamond \neg S(a)$.
Section 2

Efficient Clarification Requests and Expected Noise
Example 8

1. Every ten minutes a man gets mugged in New York. \((A)\)
2. Every ten minutes some man or other gets mugged in New York. \((F)\)
3. Every ten minutes a particular man gets mugged in New York. \((F')\)

Interpretations:

1. Every ten minutes one particular man gets mugged. \((\varphi')\)
2. Every ten minutes there is some different person who gets mugged. \((\varphi)\)
Parikh’s Model

\[ \varphi': \text{ Every ten minutes one particular man gets mugged.} \]

For \( \rho > \rho' \) the unique Pareto Nash equilibrium is:

\begin{align*}
S | & \varphi & \varphi' \\
A & F' & \\
H | & A & F & F'
\end{align*}

\begin{align*}
& \varphi & \varphi' \\
& F' & \\
& \varphi & \varphi'
\end{align*}
1. Start with a signalling game $G$.
2. Hearer can choose between many different interpretations of utterances, including their literal interpretation and their implicatures.
3. Impose pragmatic constraints and calculate the equilibria of this game which leads to a solution $(S, H)$.
4. Implicature $A \rightarrow G$ is explained if the solution $(S, H)$ satisfies

$$H(A) = G$$

i.e. if the hearer interprets $A$ as meaning $G$. 
Parikh’s Disambiguation Principle

[Parikh, 2006, p. 111]

“[. . .] if $\rho$, $\rho'$ are the shared probabilities of [S]’s intention to convey $p$, $p'$ respectively, and $b$, $b'$ are the respective marginal benefits of not conveying $p$, $p'$ explicitly then it can be shown that $p$ is communicated with certainty if and only if $\rho \cdot b > \rho' \cdot b'$.”

- In example: $b = b' = 10 - 7 = 3$. 
  $\Rightarrow \rho > \rho'$ then simple, ambiguous A means $\varphi$ with certainty!
The Challenging Example

Example 9 (Doctor’s Appointment)

Background: John is known to regularly consult two different doctors, physicians A and B. He consults A more often than B.

S: John has a doctor’s appointment at 4pm. He requests you to pick him up afterwards.

- **Observation:** S does not communicate that John is waiting at A’s practice.
- **Problem:** Most frameworks support the following pragmatic principle:

  If utterance $U$ has interpretation $\phi_1$ and $\phi_2$, and if interpretation $\phi_1$ is more probable than $\phi_2$, then the interpretation of $U$ is $\phi_1$. 
Further Problem: Out–of–Petrol Example

Example 10 (Out of Petrol)

H is standing by an obviously immobilised car and is approached by S; the following exchange takes place:

H: I am out of petrol.
S: There is a garage round the corner. (G)
   ➔ The garage is open. (R)

Problem:

1. There seems to be no structural difference between the Out–of–Petrol example and the Doctor’s Appointment example.
2. In both examples, there is no wider context which could be responsible for disambiguation.
Out–of–Petrol and Doctor’s Appointment

- **Doctor’s Appointment:**

  - at A’s
  - at B’s

- **Out–of–Petrol:**

  - $G \wedge R$
  - $G \wedge \neg R$
Needed:

1. Explanation why S’s answer in the Doctor’s Appointment example is not licensed.
2. What is the difference between the two examples?

Goal: Set up a general Model that

1. explains why the utterance in the Doctor’s Appointment example is not communicating successfully.
2. predicts that the hearer reacts with a clarification request.
Some Intuition

1. The speaker mixed up private knowledge with common knowledge.
   ⇒ Results in a mistake!

2. Hearer will react with a clarification request.

Efficiency of clarification requests:

1. Costs of clarification requests are nominal.
2. Guarantee an unambiguous response.

Efficiency implies:

1. Better to reply with clarification request than to make risky choice.
2. But does not rule out all underspecified answers!
Efficient Clarification Requests

1. Natural reaction to Ambiguity: Clarification Request.

2. Clarification request efficient: Comes with nominal costs, guarantees maximal payoff.

![Diagram showing the choice between a and b, with and without clarification requests.](image)

Left: Without CR risky choice between a and b.
Noise Resulting from Speaker’s Mistakes

- Optimal Assertions in $\sigma$: Set $\text{Op}_\sigma$
- Optimal Assertions in $\hat{\sigma}$: Set $\text{Op}_{\hat{\sigma}}$
- Result of Mixing up: Speaker may produce assertions $\text{Op}_\sigma \cup \text{Op}_{\hat{\sigma}}$ in $\sigma$. 

Diagram:

- States: $S$, $H$, $\hat{\sigma}$, $\sigma$
- Transitions: $S(A|\hat{\sigma})$, $S(A|\sigma)$
- Probabilities: $P_S$, $P_H$, $P_{\hat{\sigma}}$

Mixing up:

$\Rightarrow$ true state
Uniquely Optimal Assertions

1. $\mathcal{N}_\sigma$: Set of possibly noisy assertions; e.g.
   \[
   \bigcup \{ \text{Op}_{\hat{\sigma}} \mid \hat{\sigma} \text{ mixed up with } \sigma \}. \tag{2.3}
   \]

2. $\mathcal{B}(\sigma)$: Best actions given speaker’s knowledge.

3. Set of actions which are speaker optimal in all situations $\sigma$ in which $A$ may be asserted by a perturbed or unperturbed speaker strategy:
   \[
   \tilde{\mathcal{B}}(A) := \bigcap \{ \mathcal{B}(\sigma) \mid A \in \mathcal{N}_\sigma \}. \tag{2.4}
   \]

4. Uniquely Optimal Assertions:
   \[
   \text{UOp}_\sigma := \{ A \in \mathcal{N}_\sigma \mid \tilde{\mathcal{B}}(A) \neq \emptyset \}. \tag{2.5}
   \]
Efficient Clarification Requests and Best Response to Noisy Speaker Strategy

Assumption: Hearer can respond with efficient clarification requests

CR:
- Clarification requests have nominal costs;
- Clarification requests can force the speaker to produce an $A \in \text{UOp}_\sigma$.
- Sufficient: $\forall \sigma \cup N_\sigma \cap \text{UOp}_\sigma \neq \emptyset$.

Then, the following hearer strategy is a Best Response to assertion $A$:

1. Choose some $a \in \tilde{B}(A)$ if $\tilde{B}(A) \neq \emptyset$,
2. Else responde with a Clarification request.
This leads to a new equilibrium of unperturbed strategies:

- **Hearer Strategy given A:**
  - Choose some \( a \in \tilde{B}(A) \) if \( \tilde{B}(A) \neq \emptyset \),
  - Else responde with a Clarification request.

- **Speaker Strategy given \( \sigma \):**
  - Choose answers from \( U_{Op_\sigma} \).
Iterated Best Response and Noise

\[ S_0 \xrightarrow{\text{noise}} H_0 \]

\[ S_0 \xrightarrow{\text{best response}} H_1 \]

\[ \bar{S}_0 \xrightarrow{\text{best response}} H_1 \]

\[ \bar{S}_0 \xrightarrow{\text{best response}} H_1 \]

old equilibrium

new equilibrium
Section 3

Examples: Noisy Communication
The Out–of–Petrol–Example
The Out–of–Petrol Example

Example 11 (Out of Petrol)

\( H \) is standing by an obviously immobilised car and is approached by \( S \); the following exchange takes place:

\( H \): I am out of petrol.

\( S \): There is a garage round the corner. \((G)\)

\(+>\) The garage is open. \((R)\)
Examples: Noisy Communication

Out-of-Petrol

\[ R_l \land \neg R_r \quad \text{and} \quad \neg R_l \land R_r \]

\[ G_l \land R_l \quad \text{and} \quad G_r \land R_r \]

\[ \text{go-to-}g_1 \quad \text{and} \quad \text{go-to-}g_2 \]

\[ S \quad \text{and} \quad H \]

Anton Benz (ZAS)
Basic Observation I

Out–of–Petrol

\[ \forall \sigma : \exists F (F \in \text{Op}_\sigma \land G \triangleleft F) \Rightarrow R \triangleleft F. \quad (3.6) \]

- **G**: There is a garage round the corner.
- **R**: The garage is open.

"A \triangleleft F" read: A is a sub-formula of F.
Basic Observation II

Doctor’s Appointment
There exist support problems $\sigma, \sigma'$ such that:

$$\exists F (F \in \text{Op}_\sigma \land D \triangleleft F \land A \triangleleft F) \land \exists F (F \in \text{Op}_{\sigma'}, \land D \triangleleft F \land B \triangleleft F). \quad (3.7)$$

- $D$: Pick him up at the Doctor’s practice.
- $A$: He is waiting at A’s practice.
- $B$: He is waiting at B’s practice.

- $A \triangleleft F$: $A$ is a sub-formula of $F$. 
1. In the Out–of–Petrol but not in the Doctor’s Appointment example the utterance can be uniquely completed to an optimal answer.

2. **Conclusion:** There is a mechanism of unique optimal completion which is applied in the interpretation of the Out–of–Petrol example.

3. **Prerequisite:** The hearer $H$ has to know the possible information states of the speaker and the possible optimal answers.

4. **Hence:** The hearer has to take the speaker’s perspective into account in order to arrive at the implicature.
A Further Example

Standard Scalar Implicatures

1. All of the boys came to the party. \((F_{\forall})\)
2. Some of the boys came to the party. \((F_{\exists})\)
3. Some but not all of the boys came to the party. \((F_{\exists \neg \forall})\)
4. Not all of the boys came to the party. \((F_{\neg \forall})\)
5. None of the boys came to the party. \((F_{\neg \exists})\)

\[ F_{\exists}, F_{\neg \forall} \implies F_{\exists \neg \forall} \]
Observation

1. \( \{ F \mid \exists \sigma F \in \text{Op}_\sigma \} = \{ F_\forall, F_\exists, F_{\exists \neg \forall} \} \).

2. \( F_\exists, F_\neg \forall \not\in \{ F \mid \exists \sigma F \in \text{Op}_\sigma \} \).

3. Some and Not All are sub-forms of Some but not All:

\[
F_\exists, F_\neg \forall \triangleleft F_{\exists \neg \forall}
\]
Hypothesis

**Speaker**
1. Source of Noise: Speaker may delete *redundant* information.
2. Redundant: If all answers which contain $F_1$ also contain $F_2$, then it may happen that the speaker deletes $F_2$.

**Hearer**
If a non–optimal form $G$ can be uniquely completed to an optimal super–form $F$, then $G$ inherits the implicatures of $F$.

**Principle of Optimal Completion**
The Noise

Setting up the Model

1. Let $\mathcal{S}$ be a set of support problems which may only differ with respect to $P_\mathcal{S}$.

2. Let $\triangleleft$ be a relation on $\mathcal{F}$ which is defined as by:

   $i)$ $F \triangleleft F$, 
   $ii)$ $G \equiv F_1 \wedge F_2 \& (F \triangleleft F_1 \vee F \triangleleft F_2)$, then $F \triangleleft G$. (3.8)

3. Then set:

   $\mathcal{N}_\sigma := \text{Op}_\sigma \cup \{ F \mid \exists G \in \text{Op}_\sigma \ F \triangleleft G \}$. (3.9)
Derivation of Optimal Completion

With

\[ \mathcal{N}_\sigma := \text{Op}_\sigma \cup \{ F | \exists G \in \text{Op}_\sigma \ F \triangleleft G \} \].

It follows that:

\[ \text{UOp}_\sigma = \{ F \in \mathcal{N}_\sigma | \tilde{\mathcal{B}}(F) \neq \emptyset \} \]

\[ = \{ F \in \mathcal{N}_\sigma | \exists a \in \mathcal{A} \ \forall \sigma \ \forall G \in \text{Op}_\sigma (F \triangleleft G \Rightarrow a \in \mathcal{B}(\sigma)) \} \].

For \( S \) given, set

\[ \text{UOp} := \bigcup_{\sigma \in S} \text{UOp}_\sigma. \]
Lemma 12

Let $S$ be a set of support problems with joint decision problem $\langle (\Omega, P_H), A, u \rangle$. Assume furthermore that

1. $S$ is an expert for every $\sigma \in S$,
2. $\forall v \in \Omega \exists \sigma \in S \ P_S^\sigma(v) = 1$.

Let

$$N_\sigma := \text{Op}_\sigma \cup \left\{ F \mid \exists G \in \text{Op}_\sigma \ F \prec G \right\}.$$

Then for $F \in \text{UOp}$, $R \subseteq \Omega$ it follows that:

$$F \rightarrow R \text{ iff } \forall G \left( \exists \sigma \ G \in \text{Op}_\sigma \land F \prec G \Rightarrow G^* \cap G^+ \subseteq R \right).$$
Out-of-Petrol

Examples: Noisy Communication

\[ R_l \land \neg R_r \quad G_l \land R_l \quad go\text{-}to\text{-}g_l \quad 1 \]

\[ R_l \land R_r \quad G_l \land R_l \quad go\text{-}to\text{-}g_l \quad 1 \]

\[ R_l \land R_r \quad G_r \land R_r \quad go\text{-}to\text{-}g_2 \quad 1 \]

\[ \neg R_l \land R_r \quad G_r \land R_r \quad go\text{-}to\text{-}g_2 \quad 1 \]

\[ S \quad H \]
Another Example: Bus Tickets
[Benz, 2008]

Example 13

$H$: Where can I get the bus tickets for the excursion?

1. $S$: Ms. Müller is sitting in office 2.07. $(F_M)$
2. $S$: Bus tickets are available from Ms. Müller. $(F_H)$
3. $S$: Bus tickets are available from Ms. Müller. She is sitting in office 2.07. $(F_{MH})$

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$H$</th>
<th>$M$</th>
<th>go-to-2.07</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>$+$</td>
<td>$+$</td>
<td>1</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>$w_2$</td>
<td>$+$</td>
<td>$-$</td>
<td>0</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>$w_3$</td>
<td>$-$</td>
<td>$+$</td>
<td>0</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>$w_4$</td>
<td>$-$</td>
<td>$-$</td>
<td>1</td>
<td>$\varepsilon$</td>
</tr>
</tbody>
</table>

$\Rightarrow F_M \not\in \{ F \mid \exists \sigma F \in \text{Op}_\sigma \}, \quad F_{MH} \in \{ F \mid \exists \sigma F \in \text{Op}_\sigma \}, \quad F_M \not\prec F_{MH}$. 
The Doctor’s Appointment—Example
Example 14 (Doctor’s Appointment)

Background: John is known to regularly consult two different doctors, physicians A and B. He consults A more often than B.

**S:** John has a doctor’s appointment at 4pm. He requests you to pick him up afterwards.

+> *John is waiting at A’s practice.*
Out–of–Petrol and Doctor’s Appointment

- **Doctor’s Appointment:**

  ![Diagram of Doctor’s Appointment]

  - at A’s
  - pick–up
  - going-to-A
  - at B’s
  - pick–up
  - going-to-B

- **Out–of–Petrol:**

  ![Diagram of Out–of–Petrol]

  - $G \land R$
  - going-to-g
  - $G \land \neg R$
  - going-to-x

Examples: Noisy Communication

- Out–of–Petrol
- Doctor’s Appointment
Assumptions:

- Identify Common Ground with hearer’s information $P_H$;
- Speaker is expert, i.e. for all $\sigma$: $\exists v P_\sigma^v(v) = 1$.
- $\hat{\sigma}_1$: $P_{\hat{\sigma}_1}^\sigma (\text{John was going to A's practice}) = 1$.
- $\hat{\sigma}_2$: $P_{\hat{\sigma}_2}^\sigma (\text{John was going to B's practice}) = 1$.
- $D$ is an optimal assertion in $\hat{\sigma}_1$ and $\hat{\sigma}_2$.
- $\sigma$: $P_\sigma^\sigma (\text{John was going to A's practice}) < 1$ and $P_\sigma^\sigma (\text{John was going to A's practice}) < 1$.

Mixing up the support problems means:

$$N_\sigma = Op_\sigma \cup Op_{\hat{\sigma}_1} \cup Op_{\hat{\sigma}_2}.$$  \hspace{1cm} (3.10)

The union of the optimal answers in $\sigma$, $\hat{\sigma}_1$, $\hat{\sigma}_2$. 
The Solution

- Hearer has to choose between two actions:
  - going-to-A
  - going-to-B

- going-to-A is successful iff John was going to A’s practice;
- going-to-B is successful iff John was going to B’s practice;

Hence

$$\mathcal{B}(\hat{\sigma}_1) = \{\text{going-to-A}\} \quad \text{and} \quad \mathcal{B}(\hat{\sigma}_2) = \{\text{going-to-B}\}.$$  

and

$$\tilde{\mathcal{B}}(D) \subseteq \mathcal{B}(\hat{\sigma}_1) \cap \mathcal{B}(\hat{\sigma}_2) = \emptyset.$$  \hspace{1cm} \text{(3.11)}

Prediction: Assertion $D$ will be answered by a clarification request.
Graphical Representation

At A’s

Mixing up

At B’s

Mixing up

\[ D \in \text{Op} \]

\[ D \not\in \text{Op} \]

\[ D \in \text{Op} \]

\[ S \rightarrow H \]
Examples with Accommodation
Example 15
Smith entered the room. She was wearing a red dress.

```
female

Mixing up

female

Mixing up

male

S

H

she

female

she ∈ Op

she /∈ Adm

he /∈ Adm

he ∈ Op
```
Accommodation II

Example 16

Due to construction work this train is ending here. Please, follow the signs to the replacement bus service. We thank you for your understanding.

- Speaker wants to show that she is polite.
- Speaker implicates that the (A) need for construction work is a compelling reason for accepting the inconvenience of replacement services.
Example 16

Due to construction work this train is ending here. Please, follow the signs to the replacement bus service. **We thank you for your understanding.** (TH)

- Speaker wants to show that she is polite.
- Speaker implicates that the (A) need for construction work is a compelling reason for accepting the inconvenience of replacement services.
Accommodation II

Example 16

Due to construction work this train is ending here. Please, follow the signs to the replacement bus service. We thank you for your understanding. (TH)

- Speaker wants to show that she is polite.
- Speaker implicates that the *(A)* need for construction work is a compelling reason for accepting the inconvenience of replacement services.
The Model I

- **undr**: Hearer has understanding (because argument $A$ is compelling).
- **TH**: Thanks for understanding.

\[ \hat{\sigma} \rightarrow A \rightarrow \text{undr} \rightarrow \text{TH} \rightarrow TH \in \text{Op} \]

\[ \sigma \rightarrow A \rightarrow \text{undr} \rightarrow \text{TH} \rightarrow TH \not\in \text{Adm} \]

\[ \sigma \rightarrow A \rightarrow \neg\text{undr} \rightarrow \neg\text{TH} \rightarrow TH \not\in \text{Adm} \]
The Model II

- comp: The argument A is compelling (⇒ Hearer has understanding).
- TH: Thanks for understanding.
The Solution

- Hearer has to choose between two actions: \( comp \) and \( \neg \text{comp} \).
- \( comp \) is successful iff argument \( A \) is compelling;
- \( \neg \text{comp} \) is successful iff argument \( A \) is not compelling;

Hence:

- \( \{ \text{comp} \} = \mathcal{B}(\hat{\sigma}), \mathcal{B}(\sigma) \)
- \( \{ \neg \text{comp} \} = \mathcal{B}(\hat{\sigma}') \).

\( TH \in \mathcal{N}_\sigma, \mathcal{N}_{\hat{\sigma}} \) but \( TH \notin \mathcal{N}_{\hat{\sigma}'} \).

\( \tilde{\mathcal{B}}(TH) := \bigcap \{ \mathcal{B}(\sigma) \mid TH \in \mathcal{N}_\sigma \} = \{ \text{comp} \} \) \hfill (3.12)

Prediction: \( TH \) will implicate that the argument is compelling.
The End
Literature:


Optimal assertions and what they implicate: a uniform game theoretic approach.


Scalar implicatures in complex sentences.