On Relevance Scale Approaches*

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Abstract

We call approaches which use decision theoretic explications of Grice' relevance maxim for selecting best answers and calculating implicatures *relevance scale approaches*. In this paper we discuss these approaches with respect to the questions: Are intuitively optimal assertions identical to assertions with maximal relevance? Can classical relevance implicatures be explained by the assumption that propositions are implicated to be false exactly if they are more relevant than what the speaker has actually asserted? The answers to both questions are negative. We will show that there exists a decision theoretically defined relevance scale which the hearer can use for calculating implicatures, but we will also see that this hearer related scale is only defined after the speaker's assertion is known and, therefore, cannot be presupposed by a definition of Grice' relevance maxim.

1 Introduction

A clarification of status and satisfying formulation of the Relevance principle is one of the major desiderata of Gricean pragmatics. In the traditional formulation, the three maxims of (Quality), (Quantity), and (Relevance) can be taken together as: (QQR) *Be truthful and say as much as you can as long as it is relevant*. Applications often rely on an intuitive everyday understanding of *relevance*. That this is insufficient for a useful theory of implicatures can be seen from example as the following from Grice (1989, p. 32):

(1) A is standing by an obviously immobilized car and is approached by B, after which the following exchange takes place:
A: I am out of patrol.
B: There is a garage round the corner. (G)
$+>$ The garage is open. (H)

Grice notes that because B's remark can only be relevant if the garage is open A can conclude that H. A possible derivation of this implicature along the lines of the standard theory (Levinson 1983) could proceed as follows: let $\overline{H}$ denote the negation of H:

(2) 1. B said that G;
2. $\overline{H}$, that the garage is not open, is relevant and $G \wedge \overline{H}$ is more informative than G;
3. B observes (QQR), hence the only reason for not saying that $\overline{H}$ can be that $\overline{H}$ is false;
4. Hence H.

But, in the given context, H, that the garage is open, can also be called *relevant*. Hence, the same argument can be made with $\overline{H}$ and H interchanged, which leads to the conclusion that $\overline{H}$

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is implicated. The weak point of such derivations, clearly, is the fact that H and $\overline{H}$ can both be called relevant.

Relevance is a central notion in decision theory (Pratt, Raiffa and Schlaifer 1995). It is defined as the value of information to a decision problem of a single agent. In this paper we discuss approaches that use decision theoretic measurer as explications of Grice' notion of relevance. Such measures of relevance have been discussed e.g. by (Merin 1999), (Rooij 2004), and (Schulz and Rooij to appear). Decision theoretic relevance measures define a linear pre–order on propositions. In analogy to the standard theory of quantity implicatures, we can call these linear pre-orders *relevance scales*, and the approaches employing these scales *relevance scale approaches*. We discuss relevance scale approaches with respect to three questions: Are intuitively optimal assertions identical to assertions with maximal relevance? Can classical relevance implicatures be explained by the assumption that propositions are implicated to be false exactly if they are more relevant than what the speaker has actually asserted? Do these decision theoretic explications make inference (2) valid? In this paper, we show that all three questions receive a negative answer:

1. Negative Result: If we assume that the speaker maximises the relevance of his utterances, then no suitable decision theoretic explication of the notion of *relevance* can avoid misleading answers.

2. Negative Result: No relevance scale approach can avoid unintended implicatures in cases like the Out-of-Petrol example (1).

If the speaker cannot follow (QQR) for selecting his utterances, nor the hearer rely on it as an interpretative principle, then two fundamental properties of conversational maxims are violated. This point is confirmed by the third negative result:

3. Negative Result: The *relevance* of propositions that makes the inference in (2) valid is itself implicated information and can therefore not define a conversational maxim.

The paper divides into three parts. In the first part, Section 2, we provide a non–technical overview and discussion of our results. In the second part, Sections 3–4, we introduce the general structures that characterise relevance scale approaches. We contrast them with a game theoretic model for deriving *optimal* assertions (Benz 2006, Benz and Rooij to appear). Finally, in Sections 5–7, we prove the three negative results.

2 Overview

An essential feature of Grice' theory of conversational implicatures is the assumption that there is a joint purpose underlying every talk exchange. In the following, we concentrate on answering situations which are subordinated to a decision problem of the inquirer. The question which creates the answering situation provides an explicit shared discourse goal. We include cases where the question remains implicit as in the Out-of-Petrol example (1), where we can assume that B's assertion is an answer to the question "*Where can I buy petrol for my car*?" Implicatures will always be treated as *particularised* conversational implicatures. We restrict considerations to situations where the inquirer and the answering expert are fully cooperative, where the expert knows everything the inquirer knows, and where these facts are common knowledge. We use game and decision theory to represent these situations. Our discussion of Example (1) showed that a precise definition of relevance is necessary for a useful theory of relevance implicatures.

Game and decision theory are attractive frameworks as they allow us to explicitly study the interaction of such central pragmatic concepts as speaker and hearer's preferences, information, choice of action, and coordination of interpretation.

(Grice 1989) only gave a brief formulation to the third maxim: *Be relevant*! Grice noted that there are analogues to most conversational maxims that relate to non-linguistic joint projects. For the maxim of relevance, he illustrated this point by the following non–linguistic example (Grice 1989, p. 28): "If I am mixing ingredients for a cake, I do not expect to be handed a good book, or even an oven cloth (...)." In this situation, both persons seems to be equally competent to decide what is *relevant* and what is not; it is even more likely that the helping person is less competent than the person using the help. But in questioning and answering situations it is the inquirer who lacks information and it is the answering person, we call this person *expert*, who has the information. Grice didn't specify from whose perspective relevance has to be defined. In principle, there are two possibilities.

The main concern of our discussion is the question whether we need a game or a decision theoretic explication of Grice notion of relevance. Game and Decision theory differ in the number of decision makers which are involved in decision making. Decision theory is concerned with the decision making of single agents. Context parameters are the possible actions the decision maker can choose from, their outcomes, and the decision maker's preferences over these outcomes. Game theory is concerned with the inter-depended decision making of several agents. Hence, the general question that underlies our discussion is the question whether we need an interactional model of communication in order to explain the choice of answers and their implicatures, or whether a non-interactional model is sufficient. The latter path was taken by previous approaches to pragmatics which are based on classical game or decision theory.[1] We will show that hearer centred explications of relevance are insufficient for both, choosing useful answers and defining implicatures, even if we consider only highly favourable dialogue situations.

There seems to be a strong a priori argument in favour of the non–interactional view that stems from the calculability of implicatures. Calculability presupposes that interlocutors have access to the necessary contextual parameters. This seems to imply that the hearer must possess some measure of relevance that depends only on the semantics of utterances and common background knowledge but not on the speaker's private knowledge. Furthermore, in order to coordinate the meaning of utterances successfully, it seems necessary that the speaker uses the same definition of relevance depending on the same parameters. This reasoning leads to a hearer centred definition of relevance, and hence to a decision theoretic explication based on the hearer's, i.e. the inquirer's, decision problem. We will see that this reasoning is not conclusive. Before we address the argument of calculability, we first discuss relevance as a principle for choosing optimal answers and relevance scale explanations of implicatures.

2.1 Choosing Answers

In the following, we use the term *relevance* if we refer to decision theoretically defined measures for the value of information. "*Be relevant!*" is then interpreted as meaning that the speaker should choose answers that have a positive value of information. In order to understand its effects, we first consider the maxim of quantity. In Grice (1989) original formulation, the quantity maxim divided into two parts:

1. Make your contribution as informative as required (for the current purpose of exchange).
2. Do not make your contribution more informative than is required.

[1]See (Parikh 1992, Parikh 2001), (Parikh 1994), (Merin 1999), (Rooij 2004), (Schulz and Rooij to appear).

Grice himself noted that the second sub-maxim may be superfluous if we take relevance into account. If we see quantity and relevance in interaction, then we can simplify the first sub-maxim of quantity to "*Say as much as you can*," and restrict it by "*Say only what is relevant*." Hence, we can take the maxim of quantity and the relevance maxim together and phrase them as *Say as much as you can as long as it is relevant*. It is commonly assumed that information can be more or less relevant. The two maxims together then lead to the constraint that the speaker's utterance should provide the most relevant information possible. This principle is restricted by the maxim of quality which states that the speaker can only say what he believes to be true. Hence, we end up with a constraint (QQR) that says that the speaker can only choose the most relevant proposition which he believes to be true. Let us contrast (QQR) with a principle that combines only quality and quantity (QQ) "*Say as much as you can as long as it is true*." We consider the following example:

(3) Somewhere in the streets of Amsterdam...
I: Where can I buy an Italian newspaper?
E: At the station and at the Palace but nowhere else. (SE)
E: At the station. (A) / At the Palace. (B)

The inquirer has to decide where to go for buying an Italian newspaper. Let us assume that the answering expert knows that (SE) is true. What should he answer?[2] If we assume (QQ), then he should say everything he knows, hence only SE would conform to the maxims. But intuitively, in the given situation, A and B are equally appropriate with respect to their usefulness. This is what (QQR) predicts.

We provide an explicit model of relevance scale approaches in Section 4.2. Our representation of an answering situation σ will consist of a *decision problem* D_σ and the answering expert's expectations about the state of the world. We denote by Adm_σ the set of all propositions which the experts believes to be true. We call our representation σ a *support problem*. *Relevance* is a property of propositions and depends on a given decision problem. Propositions can be compared according to their relevance. These very general properties of relevance can be represented by real-valued functions R with two arguments, decision problems and propositions. If D_σ is a decision problem and A,B two propositions, then $R(D_\sigma,A) < R(D_\sigma,B)$ means that A is less relevant for the decision problem D_σ than B. We call these functions *relevance measures*. Hence, the set MR_σ of all maximally relevant propositions that the answering expert believes to be true consists of all those propositions in Adm_σ which are maximally relevant to D_σ with respect to a given relevance measure R. MR_σ will be defined in (4.17).

The characterisation of relevance remains very general. Special measures which have been widely tested are *sample value of information* and *utility value of information* (Pratt et al. 1995). By $EU(a)$ we denote the expected utility of performing action a given the current background knowledge. By $EU(a|A)$ we denote the expected utility of performing a after learning proposition A. Let a^* denote the action that an agent would choose before learning anything. As we assume that agents are rational, it must hold that $EU(a^*) = \max_a EU(a)$. Sample value of information A is defined as follows:

$$SVI(A) := \max_a EU(a|A) - EU(a^*|A). \tag{2.1}$$

Utility value of information A is defined as:

$$UV(A) := \max_a EU(a|A) - \max_a EU(a). \tag{2.2}$$

[2]From now on we assume that the inquirer is female and the answering expert male.

We see that $SVI(A)$ can never be negative. It can only become positive if learning A induces the agent to choose a different action. In contrast, the utility value of a proposition A can become negative. It becomes positive if the maximal expected utility after learning A is higher than the maximal expected utility before learning A. In Example (3), both measures of relevance make the correct prediction if we assume that the station and the Palace are places where Italian newspapers might be available and that both possibilities are equally probable and optimal. But if we assume that there is a slightly higher a priori expectation that there are Italian newspapers at the station, then using sample value of information would predict that only the answer B, *there are Italian newspapers at the Palace*, is relevant because only this proposition would lead to a different choice of action. If we use utility value as an explication of relevance, then the relevance principle would require the answering expert *to increase the inquirer's expectations* as much as possible. Even without example, it is clear that such a prescript must lead to misleading answers. (Benz 2006) provides a proof for support problems with completely coordinated preferences but diverging expectations. Section 5 contains an analogous result for support problems where the answering expert's expectations are derived from the inquirer's expectations by a Bayesian update. In order to prove this result we have to make stronger assumptions about relevance scales. We look at the following two examples to motivate these assumptions:

(4) There is a strike in Amsterdam and therefore the supply with foreign newspapers is a problem. The probability that there are Italian newspapers at the station is slightly higher than the probability that there are Italian newspapers at the Palace, and it might be that there are no Italian newspapers at all. All this is common knowledge between I and E. Now E learns that (N) the Palace has been supplied with foreign newspapers. In general, it is known that the probability that Italian newspapers are available at a shop increases significantly if the shop has been supplied with foreign newspapers.

(5) We assume the same scenario as in (4) but E learns this time that (M) the Palace has been supplied with *British* newspapers. Due to the fact that the British delivery service is rarely affected by strikes and not related to newspaper delivery services of other countries, this provides no evidence whether or not the Palace has been supplied with Italian newspapers.

What is of interest is the relation between the propositions N, M, and the uninformative proposition Ω, i.e. saying nothing. It is $M \subseteq N \subseteq \Omega$ and, as M has no influence on the expected success of going to the station or going to the Palace, M and Ω must be equally relevant to the underlying decision problem. In both examples, I's decision problem, i.e. her information, preferences and choices of action, are the same. This means that in both examples either N is more relevant than Ω, or it is not. But this means that N is either the most relevant answer in (5), or irrelevant in (4). Both predictions are counterintuitive. The standard relevance measures introduced in (2.1) and (2.2) e.g. both predict that N is the most relevant answer in (5).

Intuitively, in (5), E has nothing relevant to say because the most informative answer he could give has no influence on the expected utilities of any action. We can generalise this observation as follows: If A represents the expert's knowledge, and if A and Ω are equally relevant, then there must be no $A \subseteq C \subseteq \Omega$ which is more relevant than Ω. This condition would be sufficient for our purposes but, in order to have a constraint that does only depend on the inquirer's decision problem, we formulate a slightly more general *monotonicity* condition: if $A \subseteq B$, then B must not be more relevant than A. This monotonicity condition is quite strong and rules out measures like (2.1) and (2.2).

In order to explain (5) we have to assume that relevance measures are monotone. In order to explain examples like (3) we have to assume that propositions A and B which lead to identical expected utilities are equally relevant: if $\forall a EU(a|A) = EU(a|B)$, then the relevance of A and B must be equal. We call this property the *Italian newspaper property*. Finally, in situations where the answering expert thinks that saying nothing would induce the inquirer to choose a sub-optimal action there must exists some relevant answer that he can choose. The precise definitions of these properties are stated in Section 5, Def. 5.1. A fourth condition is that relevance measures must not prescribe misleading answers. In Lemma 5.2 we show that no relevance measures can satisfy all four of these properties.

As mentioned before, in principle the value of information can be determined from two perspectives, the speaker's and the hearer's. In Grice' example of handing someone ingredients for making a cake, a relevance based analogue would demand that I evaluate the ingredients according to the receiver's expectation. But why should I do so? Especially, if I am more competent than the receiver and know exactly what she is going to do with the ingredients. It would be more reasonable to deliberate first how she can handle the different ingredients and then choose those ingredient she can make the best use of. Applied to answering situations, this means that the answering expert should first find out what are the optimal actions for the inquirer, and then choose an answer that will induce her to choose one of them. In order to do this, the expert has to calculate which action the inquirer will choose after receiving the different possible answers. This leads to a game theoretic model in which the expert E calculates backward from the final outcome of I's actions a to his own decision situation where he chooses an answer A. We introduce the game theoretic model in Section 4.1. The associated set of *optimal answers* Op_σ is defined in (4.11). It is identical to the set of all non-misleading answers.

2.2 Implicatures

Relevance scale approaches typically embrace the following assumptions: (1) propositions can be ordered according to their relevance to the joint purpose of the talk exchange, (2) speaker and hearer know this order, (3) the speaker is presumed to maximise the relevance of his talk contributions, and (4) whatever is not said but would have been more relevant is implicated to be false. The fourth assumption is a consequence of the third assumption: If the speaker is presumed to maximise relevance and asserted a proposition A which is not maximally relevant, then there must have been a reason for it. Ignoring reasons as e.g. complexity or politeness, the only explanation is that A is the most relevant proposition which the speaker knows to be true. But if speakers cannot be assumed to maximise relevance, as shown before, then the relevance based account lacks a proper foundation. Moreover, we will show in Section 6 that this approach necessarily predicts undesired implicatures. The Italian newspaper example is a case where there are two propositions, "At the station" A_1 and "*At the Palace*" A_2, which must be equally relevant in order to explain why the answer A_1 does *not* implicate A_2 and vice versa. In the Out-of-Petrol example we found a case where an answer A_1 implicates some stronger proposition H_2. By merging these two examples, we get the ultimate counter example against relevance scale approaches:

(6) Somewhere in Berlin... Suppose I approaches the information desk at the entrance of a shopping centre. He wants to buy Argentine wine. He knows that staff at the information desk is very well trained and know exactly where you can buy which product in the centre. E, who serves at the information desk today, knows that there are two supermarkets selling Argentine wine, a Kaiser's supermarket in the basement and an Edeka supermarket on the first floor.
I: I want to buy some Argentine wine. Where can I get it?
E: Hm, Argentine wine. Yes, there is a Kaiser's supermarket downstairs in the basement

at the other end of the centre.

We show that no relevance scale approach can explain the (non-)implicatures in this example. We consider the following propositions:

1. A_1: There is a Kaiser's supermarket in the shopping centre.
2. A_2: There is an Edeka supermarket in the shopping centre.
3. H_1: The Kaiser's supermarket sells Argentine wine.
4. H_2: The Edeka supermarket sells Argentine wine.

A_1 and A_2 are equally relevant to the joint goal of finding a shop where I can buy Italian wine. Due to the linearity of the pre-order induced by a real valued relevance measure, all implicatures of A_2 must also be implicatures of A_1. As answering A_i implicates H_i, it follows from a relevance scale approach that answering A_1 must also implicate H_2. But the assertion that there is a Kaiser's supermarket clearly does not implicate that there is an Edeka supermarket which sells Argentine wine.

2.3 Calculability

The perhaps strongest argument in favour of relevance approaches seems to be the argument from calculability. Implicatures are part of what is communicated, hence speaker and hearer have to agree on their content, and especially the hearer has to be able to calculate them given a relevance measure that is defined relative to his local information, i.e. relative to his decision problem D_σ. If optimality of answers can only be calculated when taking into account the speaker's expectations, then, it seems, that a game theoretic approach cannot explain how the hearer is able to calculate implicatures. But this reasoning does not take into account that the hearer already knows the answer A when calculating implicatures $A +> H$. The hearer's local information must be identified with the pair (A, D_σ). As we will see, this is sufficient information for calculating implicatures in an optimal answer model. We provide two criteria which can be used for calculation. The first, Lemma 4.2, allows to calculate implicatures of the form $A +> H$ from the fact that the action a_A which the hearer will choose when learning A is optimal. The second, Lemma 7.1, is based on a relevance scale. As we saw in the previous section, relevance measures that define a linear per-order on propositions cannot, in general, be used for calculating implicatures. Lemma 7.1 makes use of the sample value of information, see (2.1), *after* learning answer A. In contrast to the relevance scale approaches discussed before, this relevance measure is defined relative to the *posterior* probability $P_I(\,.\,|A)$. It depends on the pair (A, D_σ). We will see in Section 7 that this notion of relevance makes the inference in (2) valid. Both criteria are only applicable if certain epistemic conditions are satisfied. The preconditions of the second criterion are stronger than the preconditions of the first.

In the standard theory (Levinson 1983, Ch. 3), implicatures follow logically from the semantic content of an utterance and the assumption that the speaker adheres to a number of conversational maxims. It is a defining property of conversational maxims that their knowledge is a logical precondition for determining the speaker's utterance. But, as our results show, the appropriate notion of relevance that makes inferences like (2) valid can only be measured *after* the answer has been given. The fact that a proposition is relevant is itself implicated information. Hence, maximising relevance cannot be a maxim. The proper explication of Grice' concept of relevance and the meaning of relevance in (2) cannot be the same thing. This is the third negative result about relevance measures.

The remainder of the paper contains the technical results. We first introduce our representations of answering situations, which we call *support problems*, in Section 4. Then we present two approaches to finding solutions to support problems. First we present the optimal answer approach, which is a game theoretic approach; then we characterise relevance scale approaches as described before. In sections 5–7 we show three negative results about relevance scale approaches. In Section 5 we show that relevance approaches cannot avoid misleading answers; in Section 6 we show that there are certain non-implicatures which cannot be explained by any relevance scale approach; in Section 7 we argue that the appropriate notion of relevance that makes (2) valid does not define a conversational maxim.

3 Support Problems

A decision problem is characterised by the possible states of the world, the decision maker's expectations about the state of the world, a set of actions the decision maker can choose from, and the decision maker's preferences over the outcomes of his actions. Let Ω be the set of all possible states of the world. We restrict our considerations to situations with finitely many possibilities. We represent an agent's expectations about the world by a probability distribution over Ω, i.e. a real valued function $P:\ \Omega \to \mathbf{R}$ with the following properties: (1) $P(v) \geq 0$ for all $v \in \Omega$ and (2) $\sum_{v\in\Omega} P(v) = 1$. For sets $A \subseteq \Omega$ we set $P(A) = \sum_{v\in A} P(v)$. The pair (Ω, P) is called a finite *probability space*. We represent an agent's preferences over outcomes of actions by a real valued function over action–world pairs. We collect these elements in the following structure:

Definition 3.1 *A* decision problem *is a triple* $\langle(\Omega,P),\mathcal{A},u\rangle$ *such that* (Ω,P) *is a finite probability space,* $\mathcal{A}$ *a finite, non–empty set and* $u:\ \mathcal{A}\times\Omega \to \mathbf{R}$ *a function.* $\mathcal{A}$ *is called the* action *set, and its elements* actions*.* u *is called a* payoff *or* utility *function.*

It is standard to assume that rational agents try to maximise their expected utilities. The *expected utility* of an action a is defined by:

$$EU(a) = \sum_{v\in\Omega} P(v) \times u(a,v). \tag{3.3}$$

In general, there might be several $a \in \mathcal{A}$ with $EU(a) = \max_{b\in\mathcal{A}} EU(b)$. In order to make sure that there is always a unique solution to a decision problem, we assume that the decision maker has intrinsic preferences over the actions in $\mathcal{A}$ which come only to bear if there are several optimal actions. Hence, we add a linear order $<$ to our decision problem and assume that the decision maker chooses $a = \max\{a \in \mathcal{A} \,|\, \forall b \in \mathcal{A}\, EU(b) \leq EU(a)\}$, where max is defined relative to $<$. We call $\langle(\Omega,P),(\mathcal{A},<),u\rangle$ a decision problem with *tie break* rule.

In the following, a decision problem $\langle(\Omega,P),(\mathcal{A},<),u\rangle$ represents the inquirer's situation before receiving information from an answering expert. We will assume that this problem is common knowledge. In order to get a model for the full questioning and answering situation we have to add a representation for the answering expert's situation. We only add a probability distribution P_E that represents his expectations about the world:

Definition 3.2 *A* support problem *is a five–tuple* $\langle\Omega,P_E,P_I,(\mathcal{A},<),u\rangle$ *where* (Ω,P_E) *is a finite probability space and* $\langle(\Omega,P_I),(\mathcal{A},<),u\rangle$ *a decision problem with tie break rule. We assume:*

$$\forall X \subseteq \Omega\ P_E(X) = P_I(X|K)\ \mathit{for}\ K = \{v \in \Omega \,|\, P_E(v) > 0\}. \tag{3.4}$$

Condition (3.4) implies that the expert's beliefs cannot contradict the inquirer's expectations, i.e. that for $A, B \subseteq \Omega$:

$$P_E(A) = 1 \Rightarrow P_I(A) > 0 \text{ and } P_I(A|B) = 1 \,\&\, P_E(B) = 1 \Rightarrow P_E(A) = 1. \tag{3.5}$$

For support problems $\sigma = \langle \Omega, P_E, P_I, (\mathcal{A}, <), u \rangle$ we denote by D_σ the associated decision problem $\langle (\Omega, P_I), (\mathcal{A}, <), u \rangle$ with tie break rule.

4 Solving Support Problems

A support problem represents just the fixed static parameters of the answering situation. We assume that I's decision does not depend on what she believes that E believes. Hence her epistemic state (Ω, P_I) represents just her expectations about the actual world. E's task is to provide information that is optimally suited to support I in her decision problem. Hence, E faces a decision problem himself, where his actions are the possible answers. The utilities of the answers depend on how they influence I's final choice. We find two successive decision problems:

$$\begin{array}{ccccc}
\text{Expert } E & & I \text{ decides} & & \text{Evaluation} \\
\text{answers} & & \text{for action} & & \\
\downarrow & & \downarrow & & \downarrow \\
\bullet & \xrightarrow{A} & \bullet & \xrightarrow{a} & \bullet \\
\uparrow & & \uparrow & & \uparrow \\
\text{expectations} & & \text{expectations} & & \text{utility} \\
\text{of } E & & \text{of } I & & \text{function} \\
(\Omega, P_E) & & (\Omega, P_I) & & u(a, v)
\end{array}$$

We assume that the answering expert E is fully cooperative and wants to maximise I's final success; i.e. E's payoff is identical with I's (our representation of the *Cooperative Principle*). E has to choose his answer in such a way that it optimally contributes towards I's decision. We first introduce a game theoretic solution based on (Benz 2006). Afterwards, we provide a characterisation of relevance scale approaches.

4.1 The Optimal Answers Approach

I's Decision Situation

The expected utility of actions may change if the decision maker learns new information. To determine this change of expected utility, we first have to know how learning new information affects the inquirer's beliefs. In probability theory the result of learning a proposition A is modelled by *conditional probabilities*. Let H be any proposition and A the newly learned proposition. Then, the probability of H *given* A, written $P(H|A)$, is defined by:

$$P(H|A) := P(H \cap A)/P(A). \tag{4.6}$$

This is only well–defined if $P(A) \neq 0$. In terms of this conditional probability function, the *expected utility after learning* A is defined by:

$$EU(a|A) = \sum_{v \in \Omega} P(v|A) \times u(a, v). \tag{4.7}$$

I will choose the action which maximises her expected utilities, i.e. she will only choose actions a where $EU(a,A)$ is maximal. In addition, we assume that I has always a preference for one action over the other. We represented this preference by a linear order $<$ on $\mathcal{A}$. For $A \subseteq \Omega$ we can therefore denote the inquirer's unique choice by

$$a_A := \max\{a \in \mathcal{A} \,|\, \forall b \in \mathcal{A}\; EU_I(b|A) \leq EU_I(a|A)\}. \tag{4.8}$$

E's Decision Situation

As we assume that E is fully cooperative, E has the same preferences over outcomes as I. E has to choose an answer that induces I to choose an action that maximises their common payoff. We can see E's situation as a separate decision problem where he has to choose between the answers $A \subseteq \Omega$. With a_A defined as before, we can calculate the expected utilities of the different answers as follows:

$$EU_E(A) := \sum_{v \in \Omega} P_E(v) \times u(v, a_A). \tag{4.9}$$

We add here a further Gricean maxim, the *Maxim of Quality*. We call an answer *admissible* if $P_E(A) = 1$. The Maxim of Quality is represented by the assumption that the expert E does only give admissible answers. This means that he believes them to be *true*. For a support problem $\sigma = \langle \Omega, P_E, P_I, (\mathcal{A}, <), u \rangle$ we set:

$$Adm_\sigma := \{A \subseteq \Omega \,|\, P_E(A) = 1\} \tag{4.10}$$

Hence, the set of optimal answers for σ is given by:

$$\mathrm{Op}_\sigma = \{A \in Adm_\sigma \,|\, \forall B \in Adm_\sigma\, EU_E(B) \leq EU_E(A)\}. \tag{4.11}$$

The expert may always answer everything he knows, i.e. he may answer $K := \{v \in \Omega \,|\, P_E(v) > 0\}$. From condition (3.4) it trivially follows that $EU_E(a_K) = \max_{a \in \mathcal{A}} EU_E(a)$, hence:

$$\exists A \subseteq \Omega : EU_E(a_A) = \max_{a \in \mathcal{A}} EU_E(a); \tag{4.12}$$

Let us call an answer C *misleading* iff $EU_E(a_C) < \max_{a \in \mathcal{A}} EU_E(a)$. It follows from (4.12) that Op_σ is the set of all non–misleading answers.

Calculating Implicatures from Optimal Answers

From the previous model we can derive a technical definition of what is an implicature. In the standard model (Levinson 1983), implicatures $A +> H$ follow logically from the fact that A has been uttered and the assumption that the speaker adheres to the conversational maxims. In our context, this means that implicatures follow from the fact that the utterance of A implies that it must be an optimal answer. If we assume that the speaker has true knowledge, then the truth of a proposition H follows if the speaker believes it to be true. Implicatures may depend on additional, contextually given information. This information can be represented by a subclass $\hat{\mathcal{S}}$ of support problems. The following definition applies only to propositions that can be represented as subsets of Ω, i.e. it does not capture situations where H attributes a certain belief to the speaker.

Definition 4.1 (Implicature) *Let* $\sigma = \langle \Omega, P_E, P_I, (\mathcal{A}, <), u \rangle$ *be a given support problem,* $\sigma \in \hat{\mathcal{S}} \subseteq \mathcal{S}$*. For* $A, H \in \mathcal{P}(\Omega)$*,* $A \in \mathrm{Op}_\sigma$ *we define:*

$$A +> H :\Leftrightarrow \forall \hat{\sigma} \in [\sigma]_{\hat{\mathcal{S}}} : A \in \mathrm{Op}_{\hat{\sigma}} \rightarrow P_E^{\hat{\sigma}}(H) = 1, \tag{4.13}$$

where $[\sigma]_{\hat{\mathcal{S}}} := \{\hat{\sigma} \in \hat{\mathcal{S}} \,|\, D_\sigma = D_{\hat{\sigma}}\}$.

Let $O(a)$ be the set of all worlds where a is an optimal action:

$$O(a) := \{w \in \Omega \,|\, \forall b \in \mathcal{A}\, u(a,w) \geq u(b,w)\}. \tag{4.14}$$

As a special case, we find:

Lemma 4.2 *Let* $\hat{\mathcal{S}}$ *be the set of all support problems with* $\exists a \in \mathcal{A}\, P_E(O(a)) = 1$*.*
Let $\sigma \in \hat{\mathcal{S}}$*,* $A, H \subseteq \Omega$*,* $A \in \mathrm{Op}_\sigma$*, and* $A^* := \{w \in A \,|\, P_I(w) > 0\}$*. Then it holds that:*

$$A +> H \text{ iff } A^* \cap O(a_A) \subseteq H. \tag{4.15}$$

We first show that

$$\exists a \in \mathcal{A}\, P_E^\sigma(O(a)) = 1 \,\&\, A \in \mathrm{Op}_\sigma \Rightarrow P_E^\sigma(O(a_A)) = 1. \tag{4.16}$$

Suppose $P_E^\sigma(O(a_A)) < 1$. Let a be such that $P_E^\sigma(O(a)) = 1$. Then $EU_E^\sigma(a_A) = \sum_{v \in O(a)} P_E^\sigma(v) \cdot u(a_A, v) < \sum_{v \in O(a) \cap O(a_A)} P_E^\sigma(v) \cdot u(a,v) + \sum_{v \in O(a) \setminus O(a_A)} P_E^\sigma(v) \cdot u(a,v) = EU_E^\sigma(a)$, in contradiction to $A \in \mathrm{Op}_\sigma$.
Proof of Lemma 4.2: We first show that $A^* \cap O(a_A) \subseteq H$ implies that $A +> H$. Let $\hat{\sigma} \in [\sigma]_{\hat{\mathcal{S}}}$ be such that $A \in \mathrm{Op}_{\hat{\sigma}}$. We have to show that $P_E^{\hat{\sigma}}(H) = 1$. By (4.16) $P_E^{\hat{\sigma}}(O(a_A)) = 1$ and by (3.4) $P_E^{\hat{\sigma}}(A^*) = 1$; hence $P_E^{\hat{\sigma}}(O(a_A) \cap A^*) = 1$, and it follows that $P_E^{\hat{\sigma}}(H) = 1$.
Next we show $A +> H$ implies $A^* \cap O(a_A) \subseteq H$. Suppose $A^* \cap O(a_A) \not\subseteq H$. Let $w \in A^* \cap O(a_A) \setminus H$. Let $\hat{\sigma}$ be such that $D_\sigma = D_{\hat{\sigma}}$ and $P_E^{\hat{\sigma}}(w) = 1$. As $w \in O(a_A)$, it follows that $A \in \mathrm{Op}_{\hat{\sigma}}$. Due to $A +> H$, it follows that $P_E^{\hat{\sigma}}(H) = 1$, in contradiction to $w \notin H$. ■
Both, A^* and $O(a_A)$ are known to the inquirer. The condition $A^* \cap O(a_A) \subseteq H$ is equivalent to $P_I(O(a_A) \cap H | A) = 1$. Hence, this result explains how the inquirer can calculate implicatures using her local information *after* learning answer A.

4.2 Relevance Scale Approaches

Any definition of relevance will define a real valued function R which orders propositions according to their relevance. For the question how to choose a maximally relevant answer, we can abstract away from other desirable properties of relevance measures and assume that they are general functions $R(D, .) : \mathcal{P}(\Omega) \rightarrow \mathbf{R}$ for decision problems $D = \langle (\Omega, P), \mathcal{A}, u \rangle$. Given such a relevance measure, we can define the set of maximally relevant answers. This set is restricted to the propositions which the speaker believes to be true. Let $\sigma = \langle \Omega, P_E, P_I, (\mathcal{A}, <), u \rangle$ be any support problem. Then, the set of maximally relevant answers MR_σ is given by

$$\mathrm{MR}_\sigma := \{A \in Adm_\sigma \,|\, \forall B \in Adm_\sigma\, R(D_\sigma, B) \leq R(D_\sigma, A)\}. \tag{4.17}$$

I call a theory about relevance implicatures a *relevance scale approach* iff it defines or postulates a linear pre-order $\prec$ on propositions such that an utterance of proposition A implicates a proposition H iff $A \prec \neg H$; i.e.:

$$A \prec \neg H \Leftrightarrow A +> H \tag{4.18}$$

The reasoning behind this kind of approach is roughly as follows: (1) The speaker said A; (2) $\neg H$ would have been more relevant but the speaker didn't say that $\neg H$; (3) as the speaker should say as much as he can as long as it is relevant, it follows that $\neg H$ must be false; (4) hence H.

The pre-order $\prec$ may again depend on a given decision problem. A representation by a pre-order is equivalent to a representation of preferences by a real valued function. Furthermore, implicatures may depend on commonly known background assumptions. The following definition makes these dependencies explicit. In addition, it adds the constraint that a relevance scale implicature H can only arise if the speaker is known to know whether H.

Definition 4.3 (Relevance Scale Implicature) *Let $\hat{\mathcal{S}}$ be a subset of $\mathcal{S}$ and $\sigma \in \hat{\mathcal{S}}$. Let $R(D_\sigma, .) : \mathcal{P}(\Omega) \to \mathbf{R}$ be a given relevance measure. Then, it holds in σ that $A +> H$ iff*

$$\forall \hat{\sigma} \in [\sigma]_{\hat{\mathcal{S}}} : \ (P_E^{\hat{\sigma}}(H) = 1 \vee P_E^{\hat{\sigma}}(\overline{H}) = 1) \,\&\, R(D_\sigma, A) < R(D_\sigma, \overline{H}), \tag{4.19}$$

where $[\sigma]_{\hat{\mathcal{S}}} := \{\hat{\sigma} \in \hat{\mathcal{S}} \mid D_\sigma = D_{\hat{\sigma}}\}$.

In the following sections we discuss relevance scale approaches. We will show that there are severe principled limitations to this approach. First, we will show that maximising relevance must necessarily lead to misleading answers even under extremely favourable conditions. Secondly, we will show that relevance scale approaches necessarily over-predict implicatures. These results show already that the maximisation of relevance and the derived explanation of implicatures cannot be principles of conversations which speaker and hearer are presumed to follow. As a last result, we will introduce a relevance measure that makes the relevance based reasoning in the Out-of-Petrol example valid but cannot qualify as an explication of the relevance principle.

5 First Negative Result

In this section we show that maximisation of relevance leads necessarily to misleading answers. Technically this will be achieved by comparing the set of maximally relevant answers with the set of optimal answers defined by the optimal answer approach. We know that the set of optimal answers is identical to the set of all non–misleading answers. Hence, if we know that the intersection of maximally relevant and optimal answers is empty for a given support problem, then, in this case, all maximally relevant answers must be misleading. In the previous section, we introduced relevance measures as functions $R(D, .) : \mathcal{P}(\Omega) \to \mathbf{R}$. In order to achieve our result, we have to assume that the relevance measures satisfy some additional properties. We motivated these properties in Section 2.1:

Definition 5.1 *Let $R : \mathcal{P}(\Omega) \to \mathbf{R}$ and $A \subseteq B \subseteq \Omega$. We call R a* monotone *relevance measure iff*

1. *$\forall a \in \mathcal{A}\, EU(a|A) = EU(a|B) \Rightarrow R(A) = R(B)$ (Italian newspaper);*
2. *$\forall a \in \mathcal{A}\, (EU(a|B) = \max_b EU(b|B) \Rightarrow EU(a|A) < \max_b EU(b|A)) \Rightarrow R(B) < R(A)$;*
3. *$R(B) \leq R(A)$ (monotonicity).*

The following lemma shows that there cannot be a relevance measure that satisfies these properties and avoids misleading answers.

Lemma 5.2 *For each support problem $\sigma \in \mathcal{S}$ let $R(D_\sigma, .) : \mathcal{P}(\Omega_\sigma) \to \mathbf{R}$ be a monotone relevance measure. Then, for some $\sigma \in \mathcal{S}$:*

$$\mathrm{MR}_\sigma \cap \mathrm{Op}_\sigma = \emptyset. \tag{5.20}$$

Proof: Let us assume that there are support problems σ_1, σ_2 with $D_{\sigma_1} = D_{\sigma_2}$ such that there are sets $A \subseteq C \subseteq B$ with $\forall a \in \mathcal{A}\, EU(a|A) = EU(a|B)$ and

(Σ1) $A, B, C \in Adm_{\sigma_1}$, $A, B \in \mathrm{Op}_{\sigma_1}$, and $C \notin \mathrm{Op}_{\sigma_1}$;

(Σ2) $B, C \in Adm_{\sigma_2}$, $A \notin Adm_{\sigma_2}$, $B \notin \mathrm{Op}_{\sigma_2}$, and $C \in \mathrm{Op}_{\sigma_2}$.

Suppose now that for all $\sigma \in \mathcal{S}$: $\mathrm{MR}_\sigma \cap \mathrm{Op}_\sigma \neq \emptyset$. Then, if ($\Sigma$1), it follows with monotonicity conditions 1. and 3. that $R(D_\sigma, C) \leq R(D_\sigma, B)$.[3] But if ($\Sigma$2), it follows with monotonicity condition 2. that $R(D_\sigma, C) > R(D_\sigma, B)$. Hence, the lemma follows if we can show that there are support problems σ_1, σ_2 such that (Σ1) and (Σ2) hold.

We first define the shared decision problem $D = \langle \Omega, \mathcal{A}, P_I, u \rangle$ of σ_1, σ_2. Let $\Omega = \{v_1, v_2, v_3, v_4\}$, $P_I(v_1) = P_I(v_2) = \frac{4}{14}$ and $P_I(v_3) = P_I(v_4) = \frac{3}{14}$. Let $\mathcal{A} := \{a, b\}$ with:

$$u(a, v_i) = \begin{cases} 1, \text{ for } i = 1,2 \\ 0, \text{ else} \end{cases}, \quad u(b, v_i) = \begin{cases} 0, \text{ for } i = 1,2 \\ 1, \text{ else} \end{cases}.$$

We set $A := \{v_1, v_3\}$, $B := \Omega$, and $C := \{v_1, v_3, v_4\}$. Let σ_1 be the support problem with $P_E(X) := P_I(X|A)$, and σ_2 the support problem with $P_E(X) := P_I(X|C)$. Then σ_1 has property (Σ1), and σ_2 property (Σ2). This completes the proof. ■

6 Second Negative Result

The last section showed that relevance scale approaches cannot successfully account for the choice of answers. This does not necessarily entail that they cannot successfully account for relevance implicatures. The following lemma shows that relevance scale approaches have principled problems *avoiding* certain implicatures. If a proposition A_1 is equally relevant as a second proposition A_2, then whatever A_2 implicates is also implicated by A_1. This massively overgenerates implicatures.

Lemma 6.1 *Let $\hat{\mathcal{S}}$ be the set of support problems σ where for each of the propositions $X \in \{A_1, A_2, H_1, H_2\}$ it is commonly known that E knows whether X, i.e. where it is commonly known that $P_E^\sigma(X) = 1 \vee P_E^\sigma(\overline{X}) = 1$. There exists no relevance measure $R(D_\sigma, .) : \mathcal{P}(\Omega) \to \mathbf{R}$ such that the following set of implicatures are satisfied in any $\sigma \in \hat{\mathcal{S}}$.*

1. *$A_2 +> H_2$;*
2. *not $A_1 +> \neg A_2$;*
3. *not $A_2 +> \neg A_1$;*
4. *not $A_1 +> H_2$.*

[3]For this argument it suffices that $A, B, C \in Adm_{\sigma_1}$. The lemma can be proven with the following slightly weaker conditions (IN1)–(IN3) if we take into account all conditions of (Σ1). Let $K := \{v \mid P_I(v) > 0\}$ and $K \subseteq B$. Then let (IN1) be the Italian newspaper condition from Definition 5.1, (IN2) $EU(a_K|K) > EU(a_B|K)$ then $R(D_\sigma, B) < R(D_\sigma, K)$, and (IN3) $R(D_\sigma, K) = R(D_\sigma, \Omega) \Rightarrow \forall C (K \subseteq C \subseteq \Omega \Rightarrow R(D_\sigma, C) \leq R(D_\sigma, \Omega)$.

Proof: For fixed $\sigma \in \hat{\mathcal{S}}$ and relevance measure R we write $A \prec B$ iff $R(D_\sigma, A) < R(D_\sigma, B)$. Hence, $\prec$ satisfies (4.18). We show that the above set of implicatures cannot be satisfied:

1. not $A_1 +> \neg A_2$ implies $A_1 \not\prec A_2$;
2. not $A_2 +> \neg A_1$ implies $A_2 \not\prec A_1$;
3. hence, $A_1 \approx A_2$ from lines1 and 2;
4. $A_2 +> H_2$ implies $A_2 \prec \neg H_2$;
5. hence, $A_1 \prec \neg H_2$ from lines 3 and 4;
6. not $A_1 +> H_2$ implies $A_1 \not\prec \neg H_2$, in contradiction to line 5.

■

This and the last result show already that the relevance principle cannot have the status of a conversational maxim. In the standard theory it is assumed that implicatures are derived from the assumption that the speaker adheres to the maxims. The first negative result shows that the relevance principle is not responsible for the choice of answers, hence the hearer cannot presume that the speaker is adhering to it. The second negative result shows that it necessarily produces unintended implicatures. Both results contradict what is commonly seen as a defining property of maxims, namely to be the basic principles that govern the speaker's language use and thereby being the reason that generates implicatures.

7 The Third Negative Result

In the introduction, we considered the Out-of-Petrol–Example (1) and two opposing derivations of implicatures. The validity of the inference in (2) depends on $G \wedge \overline{H}$ being more relevant than G. We show that there is a reliable explication of *relevance* that makes this inference true. In the last section we saw that the linearity of a relevance order implies that for every two equally relevant propositions A_1, A_2 it follows that whatever A_1 implicates is also implicated by A_2. We can avoid this problem if we construct a new relevance scale for each answer. We will do this using a variant of sample value of information (2.1). We have to define it relative to the addressee's *posterior* probability, i.e. relative to the probability *after* learning the answer.

Lemma 7.1 *Let* $\sigma = \langle \Omega, P_E, P_I, (\mathcal{A}, <), u \rangle$ *be a given support problem. Let* $O(a)$ *be defined as in (4.14),* $A, H \subseteq \Omega$, *and let* $\hat{\mathcal{S}}$ *be the set of support problems where* $\exists a \in \mathcal{A}\, P_E(O(a)) = 1$. *The* sample value of information K posterior *to learning A,* $SVI(K|A)$, *is defined by:*

$$SVI(K|A) := EU_I(a_{A\cap K}, A\cap K) - EU_I(a_A|A\cap K). \tag{7.21}$$

Then it holds for all $\sigma \in \hat{\mathcal{S}}$ *with* $A \in \mathrm{Op}_\sigma$ *that*

$$\text{If } \forall K \subseteq \overline{H}\; SVI(K|A) > 0, \text{ then } A +> H, \tag{7.22}$$

Proof: Let $\hat{\sigma} \in \hat{\mathcal{S}}$ be any support problem with $A \in \mathrm{Op}_{\hat{\sigma}}$ and $D_{\hat{\sigma}} = D_\sigma$. We have to show that $P_E^{\hat{\sigma}}(H) = 1$. As $D_{\hat{\sigma}} = D_\sigma$, it follows that $\forall K \subseteq \overline{H}\; SVI(K|A) > 0$ holds also for $\hat{\sigma}$. By (4.16) it holds that $P_E^{\hat{\sigma}}(O(a_A)) = 1$. By assumption, it holds that for all $v \in \overline{H}$: if $P_E^{\hat{\sigma}}(v) > 0$, then $SVI(v|A) > 0$. But $0 < EU_I(a_{\{v\}}, \{v\}) - EU_I(a_A, \{v\}) = u(a_{\{v\}}, v) - u(a_A, \{v\})$ implies $v \notin O(a_A)$. Therefore, $P_E^{\hat{\sigma}}(v) = 0$ for $v \in \overline{H}$. It follows that $P_E^{\hat{\sigma}}(\overline{H}) = 0$. ■

By definition $SVI(A|A) = 0$, hence the answer A is always the most irrelevant proposition. The relative relevance scales $SVI(\,.\,, A)$ cannot be combined to a linear order on $\mathcal{P}(\Omega)$, hence they do not allow to compare the absolute relevance of two arbitrary propositions. This violates two essential assumptions about Grice' notion of relevance.

The question whether a proposition H is relevant or not is meaningful only after an answer A is known. It follows logically from the fact that A is optimal. Hence, H's relevance is a consequence of the fact that the speaker adheres to the conversational maxims represented in the optimal answer model. But this means that it is itself implicated information. Hence, the posterior sample value of information cannot be used for defining a conversational relevance maxim. This is the third negative result.

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