Referential Chains and the Common Ground

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Abstract

In this paper we provide a description for how the iterated specific use of an indefinite NP can lead to the establishment of referential chains across dialogues and dialogue participants. We describe how they introduce discourse referents, how they are related to the common ground, and how this common ground can be represented by the dialogue participants. Of central concern is the methodological part. We combine methods known from dynamic semantics/DRT on the one side, and theories for multi–agent systems on the other. The last part provides us with a natural, and non–ad hoc model for mutual information, and the interpretation of dialogue acts.

1 Introduction

This is an investigation into the pragmatics of chains in dialogue which are established through sequences of specific uses of indefinite descriptions by different speakers, which are linked to one another, and which are related to the same object.

(1) At 7:00 am in Berlin Pankow *the 12 years old Melanie* gets bitten by a Doberman.

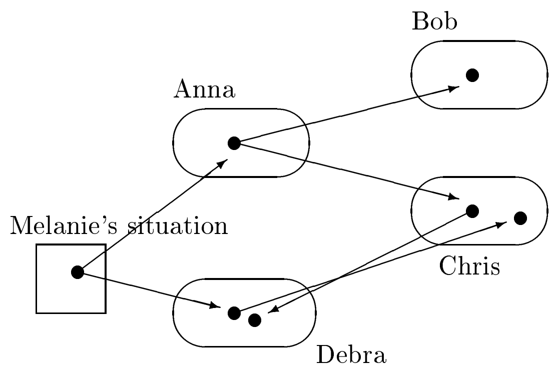
- **a)** Later, the news agency announces: "At 7:00 am in Berlin Pankow *a 12 years old girl* got bitten by a Doberman."
- **b)** Then the radio station sends the massage: "In Berlin Pankow a Doberman bit *a 12 years old girl*..."
- **c)** Ann listens to the news, and, later, she says to Bob: "Today, *a girl* has been bitten by a Doberman." Bob asks: "Do you know where *the girl* comes from?"

All the uses of *a girl* have the specific reading. They are connected to the previous uses, and are related to the same object, Melanie. This becomes clear, if we consider the meaning of the definite description *the girl* in Bob's question. It refers uniquely to Melanie, although neither Bob nor Ann need to know that.

1.1 Referential Chains

We can assume that basically each use of an indefinite NP introduces a new discourse referent into the knowledge base of the hearer. We may use here a DRT-like mechanism (Kamp & Reyle, 1993; v. Eijck & Kamp, 1997) which describes the way a hearer interprets an assertion by the speaker. What is of special interest in the case of the described chains, is that they build a connection between different dialogues, and therefore between different dialogue participants.

(2) Two passenger, Anna and Debra, observe how a Doberman bites a young girl, Melanie. The next day Anna meets Bob and Chris. They sit together, and she tells them that yesterday she saw how *a* young *girl* was bitten by a Doberman. Some weeks later, Chris meets Debra, and they come to talk about dangerous dogs. Debra tells him: "Last week, I witnessed how such a dog bit *a* little *girl*." Chris: "Oh, really! Anne told me that she too saw how a Doberman bit *a girl*."



Here, we have two dialogues, one between Anna, Chris and Bob, the other between Debra and Chris. One and the same object, Melanie, is the source for a branching chain. For subsequent dialogues, it will be necessary for the involved persons to keep track with whom they share which referent.

The problems here are closely related to the phenomena handled in the theory on *First Order Information Exchange* developed by *P. Dekker* (Dekker, 1997).

He starts out with examples like

(3) A: Yesterday, a man ran into my office, who inquired after the secretary's office.

B: Was he wearing a purple jogging suit?

A: If it was Arnold he was, and if it was somebody else he was not.

He observes that A's answer sounds strange, even if we assume that there was more than one person coming into the office, one of them Arnold. Dekker claims here that

> All natural language terms (definite and indefinite noun phrases alike), are assumed to relate to specific subjects in the information

> state of a speaker. Indefinite noun phrases which set up discourse referents in a felicitous way, must refer to specific subjects in the information state of the speaker, although they may provide no clue so as to which of his own subjects a speaker refers to. (Dekker & van Rooy, 1999)

Dekker and van Rooy developed this approach further to handle belief attributions. The meaning of a discourse like "*Melanie is a 12 years old girl. Chris believes that some dog bit her.*" can be described by a DRS like:

$$
\begin{array}{|c|}
\hline
x, y, yx, yz \\
\hline
\\
\text{Melanie}(x) \\
\text{twelve-years-old-girl}(x) \\
\text{Chris}(y) \\
y : \begin{array}{|c|}
\hline
x, z \\
\hline
\text{dog}(z) \\
\text{bit}(z, x) \\
\hline
\end{array} \\
x \text{ is the source of } yx \\
\\
\hline
\end{array}
$$

This framework can be developed straight forward to be able to describe the building of chains across dialogues and dialogue participants. We will do this in a framework of *Multi–Agent Systems*, see (Fagin e.al., 1995). I.e. we will describe the dialogues and the updates of knowledge bases of the participants as games. This has the advantage that we can exploit standard techniques to define the information an agent has in a certain dialogue situation in a possible worlds framework. Thereby, we can work with the usual definition of mutual knowledge. Multi–agent systems allow for a representation of the environment, and effects of actions on this context. We will make referential chains part of the environment, and describe how specifically used indefinites establish such chains. The *source*–relation, which is a primitive relation in the theory of Dekker and van Rooy, is thereby defined through the rules of the dialogue games.

1.2 Definite Reference and the Common Ground

The relation between established chains and the use of definite descriptions is of special interest, because it forces us to investigate how discourse referents are connected to the *common ground*.

It has become usual to identify the common ground with what is *mutually known* by the dialogue participants. The relation between the referential use of definite descriptions and mutual knowledge has been extensively studied in (Clark & Marshall, 1981). A major point was to show that, indeed, mutual knowledge — or common knowledge — of a fact $\varphi(x)$ is necessary to refer with *the* φ to a given object a. Mutual knowledge of a fact φ is identified with a conjunction of all sentences of the form:

X_1 knows that X_2 knows that X_3 knows that ... that X_n knows that φ,

where the X_n's are dialogue participants and n ranges over all natural numbers. For a *visual situation* use, it can be shown that the referential use of a definite description def $x.\varphi(x)$ is successful if the object referred to is the only one for which it is common knowledge that it has the property φ, see (Benz, 1999). In this case, the participants know the objects they talk about. But in case of an *anaphoric* use this is no more the case.

The referent of a definite description is an object in the real source situation which is normally not known to the discourse participants. But the uniqueness condition connected with definiteness is not a condition on these real objects but on the common discourse referents and their properties. E.g. assume that in Example **(2)** two reporter, A and B, listen to the last dialogue between Debra and Chris. It is not possible to continue with:

(4) A to B: We can make an interview with *the girl*.

Although there is one, and only one girl (φ) in the situation talked about, it is not possible for B to interpret the definite def$x.\varphi(x)$ as there are two common discourse referents u_i with $\varphi(u_i)$.

That the anaphoric referential use of a definite is sensitive only to the *common* discourse referents can be seen in the following example.

(5) At 7:00 am Anna observes how a Doberman bites the young girl Melanie. Some minutes later the Doberman attacks also another young girl, Stefanie. This time, it is Debra who observes it.

- **a)** Then Anna meets Bob and Chris and tells them that she has seen how a Doberman attacked a young girl.
- **b)** The next day, Debra meets Bob, and she tells him that the dog attacked also another young girl.
- **c)** Later, she meets also Chris and tells him the same.
- **d)** Chris, who does not know that Bob knows already the whole story, meets Bob again and says to him: "*The young girl* was not the only one who was attacked by the dangerous Doberman."

The use of *the young girl* by Chris is felicitous although both of them know that there have been two young girls who were attacked by the Doberman. Only one of them is available through a common referent. There arises the problem that the referential use of a definite description def $x.\varphi(x)$ is felicitous although there is no real object a such that $\varphi(a)$ is mutually known.

We split the presentation of our theory in two parts. In the first part we provide a more informal description including some examples. In the second part we add the mathematics.

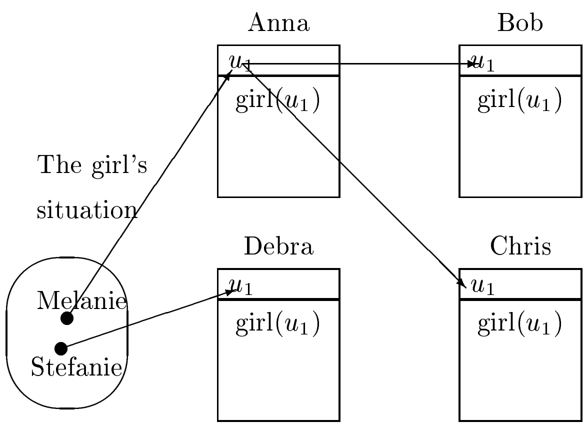
Part I
The Description of the Theory

In this part we provide for an informal outline of our theory. First we will describe a dialogue fragment with observations and assertions where we can represent referential chains. This is done by using theories of multi–agent systems. Then we ask how to interpret definite descriptions. We thereby introduce the notion of a common DRS. Finally, we consider the representation problem. We define a new dialogue fragment where participants also maintain representations for the common DRSes.

2 A Dialogue Fragment with Referential Chains

2.1 The Basic Idea for the Representation of Dialogue Situations

The following picture shows (a part of) the situation of Example **(5)** after a) where Anna told Bob and Chris that a young girl was bitten by a Doberman.



We represent this situation in a tuple of DRSes, one DRS for each dialogue participant, a first order model $\mathcal{M}$ for the situation talked about, and a relation $\rightarrow$ which tells us which discourse referent is chained to which other referent, or object.

$\mathcal{M}$	$\rightarrow$	Anna *An*	Debra *De*	Bob *Bo*	Chris *Ch*
Mel Ste	$Mel \rightarrow u_1^{An}$ $Ste \rightarrow u_1^{De}$ $u_1^{An} \rightarrow u_1^{Bo}$ $u_1^{An} \rightarrow u_1^{Ch}$	u_1 girl(u_1)	u_1 girl(u_1)	u_1 girl(u_1)	u_1 girl(u_1)

We will consider only DRSes of a very simple form, i.e. the set of conditions Con_D for a DRS D will contain only first order formulas. Hence, each DRS can be identified with the conjunction of all these formulas. The free variables are the discourse referents in $\mathcal{U}_D$. We will allow only for dialogue where each referent is connected by a chain to exactly one object in the situation talked about. Hence, the chains define for each discourse participant a an assignment function f_a for the free variables of the first order formula associated to D_a. This allows us to define the *truth* of this formula in the usual way. We write $(M, f_a) \models D_a$. As a first order formula we can identify the *meaning* of a DRS D with the set of all pairs $(\mathcal{N}, f)$ such that (1) $\mathcal{N}$ is a first order model for a possible described situation, (2) f is an assignment function for the referents in $\mathcal{U}_D$, and (3) $(\mathcal{N}, f) \models D$. This allows us to compare our approach with the usual semantic interpretation, e.g. (Kamp & Reyle, 1993), or related dynamic interpretations, e.g. (Dekker, 2000).

We will see later that we have to include additional information in the local states of dialogue participants. We will assume that the participants remember in which dialogue acts they have been involved, and which objects they have observed. We need observations in order to be able to start with situations where no participant has any knowledge about the situation talked about. Hence, we have to add to the local states of participants two entries. One consisting in a sequence of dialogue acts, the other one in a function which maps some of the referents to observed objects.

2.2 The Representation as Multi–Agent System

We describe the relevant fragment of possible dialogues as special cases of *Multi–Agent Systems*. Here, we follow the theory developed in (Fagin e.al., 1995). Within this framework we are able to describe the building of chains across dialogues and dialogue participants. I.e. we will describe the dialogues and the updates of knowledge bases of the participants as games. This has the advantage that we can exploit standard techniques to define the information an agent has in a certain dialogue situation in a possible worlds framework, and we get the usual definition of mutual knowledge.

The multi–agent system consists of the following components

1. A set $\mathcal{S}$ of *global states*.
2. A set ACT of possible dialogue acts.
3. A function P which tells us which dialogue acts can be performed in which dialogue situations.
4. A transition operation $\tau : \mathrm{ACT} \times \mathcal{S} \longrightarrow \mathcal{S}$ which models the effect of the performance of a dialogue act in a certain situation.
5. A set of initial dialogue situations S_0.

Dialogues are then identified with the set $\mathcal{G}$ of all sequences

$$\langle s_0, \mathtt{act}_0, \dots, \mathtt{act}_{n-1}, s_n \rangle$$

where s_0 is an initial dialogue situation, and:

- $\mathtt{act}_i \in P(s_i)$, i.e. $\mathtt{act}_i$ is possible in s_i.
- $s_{i+1} = \tau(\mathtt{act}_i, s_i)$.

We denote by $\mathcal{S}(\mathcal{G})$ the set of all global states which may arise as a possible dialogue situation, i.e. all situations which belong to a $G \in \mathcal{G}$.

We describe a dialogue fragment with assertions and observations which allows the building of chains by the use of indefinite NPs.

1. We assume that basically each use of an indefinite NP introduces a new discourse referent into the knowledge base of the hearer.
2. We use here a DRT–like mechanism which describes the way a hearer interprets an assertion by the speaker.

3. In order to be able to start with *empty* DRSes for the participants, we include *observations* in the set of possible dialogue acts.

4. In addition we assume that each participant remembers the dialogue acts he has been involved in. This is necessary in order to have some information about the dialogue in the local states of the participants.

2.2.1 Global States

A global dialogue state consists of the *local* states of the participants DP $= \{1, \ldots, m\}$, and the state of the environment. They are represented by tuples

$$\langle \mathcal{M}, \rightarrow, D_1, \ldots, D_m \rangle$$

$\mathcal{M}$: A first order model which describes the situation talked about.

D_a: A simple DRS extended with information (1) about the dialogue acts where the participant a was involved, and (2) about the real objects he has observed.

$\rightarrow$: A relation between objects and subjects, or subjects and subjects, where a subject is a pair $\langle a, u \rangle$ of a participant a and a discourse referent u.

We write u^a for $\langle a, u \rangle$. If a new discourse referent is introduced into a DRS, then $\rightarrow$ will connect this referent to it's *source*. A DRS D_a will have the form:

Participant a
$\mathcal{U}_{D_a}$
Con_{D_a}
Dialogue Acts where Participant a was involved
Objects he has observed

The knowledge of a participant in a multi–agent system is defined as the set of all global states where his local state is the same as in the real one. Hence, if he should have knowledge about others, there must be entries in his local state which contain information about them. This is the reason to include the sequences of dialogue acts where a participant has been involved. As will become clear from the definition of these acts in the next subsection, a participant therefore knows the group of involved participants, and the direct source of the information.

2.2.2 The Possible Dialogue Acts

We allow for three actions an agent can perform:

- $\texttt{send}(a, H, D, l)$,
- $\texttt{get}(a, H, D)$,
- $\texttt{observe}(H, D, l)$.

$\texttt{send}(a, H, D, l)$: Represents an assertion of speaker a with co–present addressees H. D is a DRS, which is the result of translating the speaker's utterance into a DRS by standard techniques, e.g. (Kamp & Reyle, 1993). l is an injective function which relates the discourse referents in $\mathcal{U}_D$ to the subjects in D_a.

$\texttt{get}(a, H, D)$: An action performed on the local states of the dialogue participants $b \in H$ who are the addressees of an assertion with content D and speaker a.

$\texttt{observe}(H, D, l)$: If performed by an agent a, it means that he is a member of a (co–present) group H which observes some fact represented by a DRS D. l is an injective function which relates the referents in $\mathcal{U}_D$ to real objects in the universe $|\mathcal{M}|$.

These actions can be performed as parts of *joined* actions. They can be identified with sequences $(\texttt{act}_a)_{a \in \mathrm{DP}}$, where $\texttt{act}_a$ is one of the three (local) actions defined before. There are two sorts of joined actions:

- $\texttt{act}_a = \texttt{observe}(H, D, l)$ with fixed H, D, l for $a \in H$, and $\texttt{act}_a = \perp$ for $a \notin H$.
- $\texttt{act}_a = \texttt{send}(b, D, H, l)$ for $a = b$, $\texttt{act}_a = \texttt{get}(H, D, l)$ for $a \in H$, and $\texttt{act}_a = \perp$ for $a \notin H \cup \{b\}$, D, H fixed.

2.2.3 When do we Allow to Perform a Joined Action?

We have to say which joined actions can be performed in a global state s. We assume that a joined observation is always possible, if the observed facts really hold. If the joined action represents an assertion with $\texttt{send}(b, H, D, l)$, then it should be a possible action in s, iff $D_b^s \trianglelefteq_l D$. Where $D \trianglelefteq_l D'$ holds between DRSes D, D', iff l is a function from $\mathcal{U}_{D'}$ to $\mathcal{U}_D$ such that for all condition $\varphi \in Con_{D'}$ $\varphi/_s l$ is an element of Con_D where $\varphi/_s l$ denotes the formula, where the free variables in φ are replaced by their l–values. E.g. let D_b^s and D be as follows where D_b^s is on the left and D on the right side:

u_1, u_2, u_3, u_4
$\mathrm{Dob}(u_1)$
$\mathrm{girl}(u_2)$
$\mathrm{bit}(u_1, u_2)$
$\mathrm{Dob}(u_3)$
$\mathrm{girl}(u_4)$
$\mathrm{bit}(u_3, u_4)$

u_1, u_2
$\mathrm{Dob}(u_1)$
$\mathrm{girl}(u_2)$
$\mathrm{bit}(u_1, u_2)$

Then b is licensed to make an utterance with content D where the referents u_1 and u_2 in $\mathcal{U}_D$ are related either to u_1 and u_2 or u_3 and u_4 in the left DRS.

The relation $D_b^s \trianglelefteq_l D$ is essentially *Dekker's* (Dekker, 1997) condition for the licensing of first order formulas. It says that a speaker can only make an assertion represented by a DRS D, iff he has *own information* about the objects referred to. This does not mean that he must be able to identify the objects related to the discourse referents in his DRS. In our context, this condition implies that the speaker can make only *true* assertions. This leads to the following two conditions:

$\texttt{observe}(H, D, l)$: Is always possible if $(\mathcal{M}, l) \models D$.

$\texttt{send}(b, H, D, l)$: It should be a possible action in a global state s, iff $D_b^s \trianglelefteq_l D$.

A (local) act $\texttt{get}(b, H, D)$ can occur only as part of a joined action together with one $\texttt{send}(b, H, D, l)$ action. It will always be performed, iff a `send`–act is performed.

For global states s we denote by $P(s)$ the set of joined actions which can be performed in this situation.

2.2.4 The Effects of Performing an Action

What are the effects of performing a joined action $(\texttt{act}_a)_{a \in \mathrm{DP}}$?

Acts with $\texttt{send}(b, H, D, l)$ These joined acts change only the local states of b and the members of the group H.

- $\texttt{send}(b, H, D, l)$ does not change the state of the speaker b except that he remembers that he has performed this action, i.e. we assume that D_b has a component Act_{D_b} such that for the new state D'_b we get $Act_{D'_b} = Act_{D_b}{}^\wedge\langle\texttt{send}(b, H, D, l)\rangle$.
- The adjoined act $\texttt{get}(b, H, D)$ should result in a merge of D_a and D for $a \in H$, and in an extension of Act_{D_a} to $Act_{D_a}{}^\wedge\langle\texttt{get}(b, H, D)\rangle$.

There is some freedom in defining this merge. We may assume that it introduces for each referent in D a new referent into $\mathcal{U}_{D_b}$, and adds the conditions of Con_D to Con_{D_b} where the old variables are replaced accordingly. $\rightarrow$ belongs to the environment, and an assertion of the form $\texttt{send}(b, H, D, l)$ has the effect that new chains are added to $\rightarrow$. They will connect the newly introduced subjects in the addressee's DRSes with subjects in the speaker's DRS.

Common Observations In order to be able to start in our examples with *empty* representations, we consider also acts of joined observations. If a is a member of a group H, then an observation $\texttt{observe}(H, D, l)$ should have the effect that D is merged to his old information D_a in such a way that new discourse referents are introduced for objects which he has not jet observed. We assume that D_a has a fourth component Obs_{D_a} which is an injective function relating referents in $\mathcal{U}_{D_a}$ to objects in $|\mathcal{M}|$, i.e. a remembers which objects he has observed.

2.3 An Example

The precise definitions will be provided in the second part of the paper where we present the mathematics of our approach. We don't go into greater details here.

We reconsider Example **(5)**. First Anna observes that a Doberman bites Melanie, and Debra observes that he bites Stefanie. Let D denote the DRS

$$\begin{array}{|c|}\hline u_1, u_2 \\ \hline \mathrm{Dob}(u_1) \\ \mathrm{girl}(u_2) \\ \mathrm{bit}(u_1, u_2) \\ \hline \end{array}$$

We assume that the content of all asserted sentences can be represented by D. The two actions

$$\mathtt{act}_1 = \mathtt{observe}(\{An\}, D, l_1), \mathtt{act}_2 = \mathtt{observe}(\{De\}, D, l_2),$$

with $l_1(u_1) = Dob = l_2(u_1), l_1(u_2) = Mel, l_2(u_2) = Ste$, result in the global state:

$\rightarrow$	An	De	Bo	Ch
$Dob \rightarrow u_1^{An}$	u_1, u_2	u_1, u_2	$\emptyset$	$\emptyset$
$Mel \rightarrow u_2^{An}$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$	$\emptyset$	$\emptyset$
$Dob \rightarrow u_1^{De}$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$	$\emptyset$	$\emptyset$
$Ste \rightarrow u_2^{De}$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$	$\emptyset$	$\emptyset$
	$\mathtt{act}_1$	$\mathtt{act}_2$		
	l_1	l_2		

(a) Then Anna meets Bob and Chris and tells them that she has seen how a Doberman attacked a young girl.

We assume, she said to them: *A Doberman bit a girl.* We can represent this assertion by the actions

$$\mathtt{act}_3 = \mathtt{send}(An, \{Bo, Ch\}, D, l_3), \mathtt{act}_4 = \mathtt{get}(An, \{Bo, Ch\}, D),$$

with $l_3(u_1) = u_1^{An}, l_3(u_2) = u_2^{An}$. It results in the global state:

$\rightarrow$	An	De	Bo	Ch
$Dob \rightarrow u_1^{An}$	u_1, u_2	u_1, u_2	u_1, u_2	u_1, u_2
$Mel \rightarrow u_2^{An}$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$
$Dob \rightarrow u_1^{De}$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$
$Ste \rightarrow u_2^{De}$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$
$u_1^{An} \rightarrow u_1^{Bo}$	$\mathtt{act}_1$	$\mathtt{act}_2$	$\mathtt{act}_4$	$\mathtt{act}_4$
$u_2^{An} \rightarrow u_2^{Bo}$	$\mathtt{act}_3$	l_2	$\emptyset$	$\emptyset$
$u_1^{An} \rightarrow u_1^{Ch}$	l_1			
$u_2^{An} \rightarrow u_2^{Ch}$				

(b) The next day, Debra meets Bob, and she tells him that the dog attacked also another young girl.

We again assume that she tells him: *A Doberman bit a girl.* We can represent this situation by the actions

$$\mathtt{act}_5 = \mathtt{send}(De, \{Bo\}, D, l_5), \mathtt{act}_6 = \mathtt{get}(De, \{Bo\}, D),$$

with $l_5(u_1) = u_1^{De}, l_5(u_2) = u_2^{De}$. It results in the global state:

$\to$	An	De	Bo	Ch
$Dob \to u_1^{An}$	u_1, u_2	u_1, u_2	u_1, u_2, u_3, u_4	u_1, u_2
$Mel \to u_2^{An}$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$
$Dob \to u_1^{De}$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$
$Ste \to u_2^{De}$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$
$u_1^{An} \to u_1^{Bo}$	$\mathtt{act}_1$	$\mathtt{act}_2$	$\mathrm{Dob}(u_3)$	$\mathtt{act}_4$
$u_2^{An} \to u_2^{Bo}$	$\mathtt{act}_3$	$\mathtt{act}_5$	$\mathrm{girl}(u_4)$	$\emptyset$
$u_1^{An} \to u_1^{Ch}$	l_1	l_2	$\mathrm{bit}(u_3, u_4)$	
$u_2^{An} \to u_2^{Ch}$			$\mathtt{act}_4$	
$u_1^{De} \to u_3^{Bo}$			$\mathtt{act}_6$	
$u_2^{De} \to u_4^{Bo}$			$\emptyset$	

(c) Later, Debra meets also Chris and tells him the same.

We can represent this situation by the actions

$$\mathtt{act}_7 = \mathtt{send}(De, \{Ch\}, D, l_7), \mathtt{act}_8 = \mathtt{get}(De, \{Ch\}, D),$$

with $l_7(u_1) = u_1^{De}, l_7(u_2) = u_2^{De}$. It results in the global state:

$\to$	An	De	Bo	Ch
$Dob \to u_1^{An}$	u_1, u_2	u_1, u_2	u_1, u_2, u_3, u_4	u_1, u_2, u_3, u_4
$Mel \to u_2^{An}$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$	$\mathrm{Dob}(u_1)$
$Dob \to u_1^{De}$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$	$\mathrm{girl}(u_2)$
$Ste \to u_2^{De}$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$	$\mathrm{bit}(u_1, u_2)$
$u_1^{An} \to u_1^{Bo}$	$\mathtt{act}_1$	$\mathtt{act}_2$	$\mathrm{Dob}(u_3)$	$\mathrm{Dob}(u_3)$
$u_2^{An} \to u_2^{Bo}$	$\mathtt{act}_3$	$\mathtt{act}_5$	$\mathrm{girl}(u_4)$	$\mathrm{girl}(u_4)$
$u_1^{An} \to u_1^{Ch}$	l_1	$\mathtt{act}_7$	$\mathrm{bit}(u_3, u_4)$	$\mathrm{bit}(u_3, u_4)$
$u_2^{An} \to u_2^{Ch}$		l_2	$\mathtt{act}_4$	$\mathtt{act}_4$
$u_1^{De} \to u_3^{Bo}$			$\mathtt{act}_6$	$\mathtt{act}_8$
$u_2^{De} \to u_4^{Bo}$			$\emptyset$	$\emptyset$
$u_1^{De} \to u_3^{Ch}$				
$u_2^{De} \to u_4^{Ch}$				

3 The Uniqueness Condition for Definite Descriptions

For multi–agent systems it is usual to identify the *knowledge* of an agent a in a situation s relative to $\mathcal{S}(\mathcal{G})$ with the set of all situation which are *indiscernible* from s. Two situations are indiscernible for an agent a, iff his local states are identical for both situations. This allows us to include the information of agents about the global state, and their information about others into our model. We either may use *Kripke*–structures, see (Fagin e.al., 1995), or develop our theory along the lines of (Gerbrandy & Groeneveld, 1997) as a *possibility* approach. Both descriptions provide us with (equivalent) representations $CG_w(H)$ of the *common ground* for a possibility w and a group H. It is a set of accessible possible dialogue situations and contains all possibilities which are possible according to the knowledge of one participant, possible according to the knowledge a participant can have according to the knowledge of an other participants, etc.

Hence, the general apparatus for multi–agent system provides us with a natural representation of the mutual information of dialogue participants. But in view of our problem to explain the anaphoric referential use of a definite description we need a representation which provides us more directly with information about which subjects with which properties are common. For this reason we introduce the notion of a *common DRS*.

First, that a DRS is *joined* should mean that it can be embedded into all the DRSes representing the knowledge of the members of the group H in such a way that the images of one referent are all connected to each other via a common source. Then, a DRS is *mutually* joined if it is joined, everybody knows that it is joined, everybody knows that everybody knows that it is joined etc. By an iteration of a suitable condition we get an intuitive definition of a *common* DRS.

More specific, D is a joined DRS for a group H and state $s \in \mathcal{S}(\mathcal{G})$, iff there is a family of injective functions $(l_a)_{a\in H}$ such that for all $a \in H$ $D^s_a \trianglelefteq_{l_a} D$, and $\forall u \in \mathcal{U}_D\ \exists x\ \forall a \in H\ x \to_r l_a(u)$, where $\to_r$ denotes the reflexive closure of $\to$. l_a embeds D into the local DRS D_a of participant a. This is done for all participants in the group H. That a discourse referent u in $\mathcal{U}_D$ is intuitively *joined* is modelled by the condition that for all participants $a \in H$ $l_a(u)$ is connected via $\to_r$ to the same subject. We need the reflexive closure $\to_r$ of $\to$ because one of the agents b in the group H may have been a speaker who introduced $l_a(u)$ to the rest of them. In this case $l_b(u)$ must be connected to itself. In order to restrict the possible size of a joined DRS we add the condition that for all $u, u' \in \mathcal{U}_D$, $u \neq u'$, there is at least one $a \in H$ such that $l_a(u) \neq l_a(u')$.

In order to define what it means that a DRS D is mutually joined we must iterate the condition of joinedness in a suitable way. Hence, D must be a joined DRS relative to a family $(l_a)_{a\in H}$. Then, every member b of H must know that D is joined. Hence, for all $b \in H$ and for all v which are possible for b in w there exists a family $(l^v_a)_{a\in H}$ such that D is joined in v relative to $(l^v_a)_{a\in H}$. In addition, we require that $l^v_b = l^w_b$, and that all the $l^v_a(u)$ are chained to the 'same' subject as in w, i.e.:

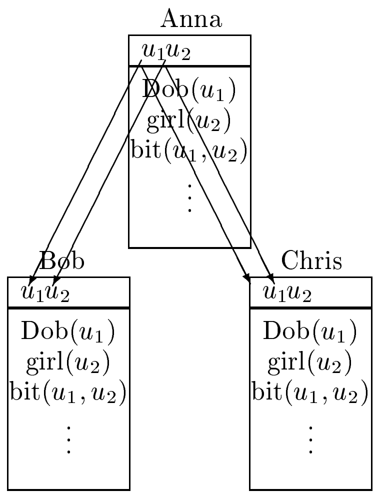
- If all $l^w_a(u)$ have been chained to $l^w_b(u)$, then in v we should also find that all $l^v_a(u)$ are chained to $l^v_b(u)$.
- If all $l^w_a(u)$ have been chained to a subject $x \neq l^w_b(u)$, then in v we should find that the $l^v_a(u)$s are again chained to an $x \neq l^v_b(u)$.

By the iteration of this condition we get the definition of a *common* DRS $C_w(D, H)$ for a group H in a situation w (Def. 8.2).

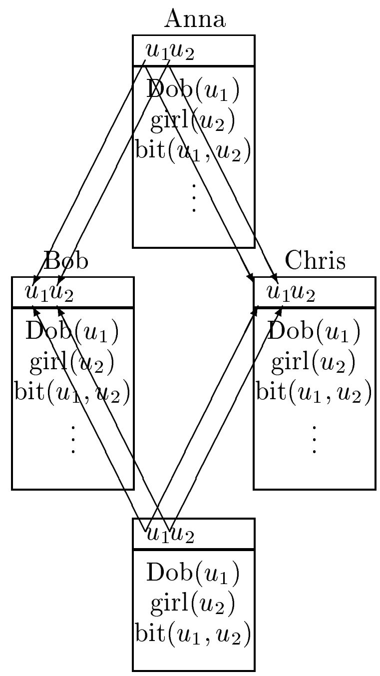
We consider again Example **(5)**. For the last global state described on page 11, which represents the situation after (c), we find that the following DRS D is a maximal common DRS for the group $H_1 = \{An, Bo, Ch\}$.

u_1, u_2
Dob(u_1)
girl(u_2)
bit(u_1, u_2)

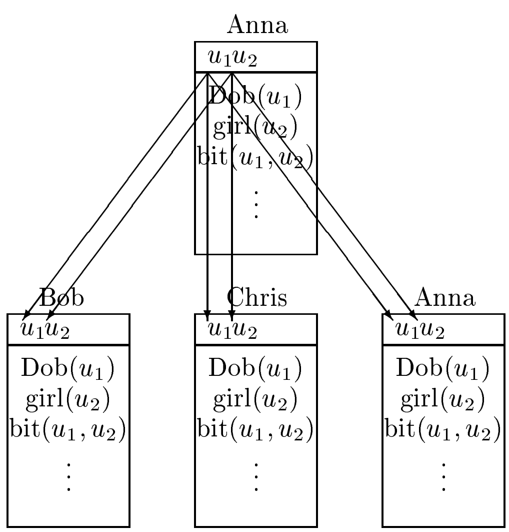
Anna told them in situation a) that *A Doberman bit a girl.* Hence, the resulting structure is a substructure of all subsequent global situations. Therefor, we see that D is a joined DRS. As embeddings we can use the identity functions, i.e. for all $a \in H_1$ we set $l_a(u_i) = u_i$.



Anna's utterance of *A Doberman bit a girl* produced this structure, which is a substructure of all subsequent global states. It allows for the embedding depicted on the right side.



It is clear that they are embeddings, and we can check that for all a and $i = 1, 2$ $l_a(u_i)$ is chained to the same subject, namely u_i^{An}. This makes clear why we needed the reflexive closure in our definition: it allows us to chain the referents of the speaker to it's own referents.



If we want to show that it is a mutual DRS, then we have to look at the local states of the participants and their knowledge about each other. Let us consider e.g. the local state of Bob. He remembers that he was involved in the joined act defined by $\texttt{act}_3$ and $\texttt{act}_4$, as $\texttt{act}_4 = \texttt{get}(An, H_1, D)$ was concatenated to Act_{Bo}, a part of his local state. But the general constraints for our dialogue fragment imply that for any dialogue where $\texttt{act}_4$ is part of Bobs local state $\texttt{act}_4$ is also a part of the local states of all other members of H_1, i.e. of Chris local state. Furthermore, the rules imply that all referents which have been introduced into Bobs local state by $\mathtt{act}_4$ are chained to referents in the speaker's, i.e. Anna's, local state. The same follows for Chris. Hence, it follows that Bob *knows* that D is a joined DRS. This reasoning applies in the same way for Chris, and in a similar way for Anna. Hence, all know that D is a joined DRS.

The definition of indiscernability implies that all situations that are possible according to the knowledge of an agent possess the same local state for him. Hence, it follows that for all these possible global states D is a joined DRS. Furthermore, it shows that the embedding functions can be chosen in the right way. An iteration of this sort of reasoning shows then that D is a common DRS.

Note, that D is also a common DRS for the groups $\{An, Bo\}$, $\{An, Ch\}$, and $\{Bo, Ch\}$ but not for $\{De, Bo, Ch\}$. It is also a maximal common DRS for the group $\{An, Bo, Ch\}$. This follows only because we restricted the way how a DRS can be embedded. If we do not include the conditions that the l_as must be injective and that for all $u, u' \in \mathcal{U}_D$, $u \neq u'$ there is at least one $a \in H$ such that $l_a(u) \neq l_a(u')$, then any DRS which results as an iterated merge of D to itself, i.e. $D \oplus D$ or $(D \oplus D) \oplus D$ etc., would have been a common DRS.

4 The Representation Problem

The last section provided us with a reasonable description of a *common* DRS. But how can the participants have access to this DRS? The most intuitive way seems to be that they keep track of the discourse referents which have been introduced to each group, and about the properties of those referents. I.e. a participant will not only update his own DRS, if he gets some new information, but he will also update a DRS representing the knowledge of the group which *commonly* got this information. This leads to an extension of the local states. We add for each participant a and for each group $H \subseteq \mathrm{DP}$, where he is a member of this group, representing DRSes $D_{a,H}$. In the same way as in the last section we can describe the update operations connected to the possible local acts $\mathtt{send}(a, H, D, l)$, $\mathtt{get}(a, H, D)$ and $\mathtt{observe}(H, D, l)$ for global states with representations. Together with the function P, which specifies which actions are possible in a certain situation, this will lead to a new set of possible dialogues $\mathcal{G}^+$ for the same sequences of actions.

The following figure describes (part of) the local state of Bob in Example 5 after his talk with Debra (2). The first column represents his total knowledge about the biting situation, the second his protocol for what he heard in common with Anne and Chris, and the third for what he has in common with Debra.

Bo	$\{Bo, An, Ch\}$	$\{Bo, De\}$
u_1, u_2, u_3, u_4	u_1, u_2	u_1, u_2
Dob(u_1) girl(u_2) bit(u_1, u_2) Dob(u_3) girl(u_4) bit(u_3, u_4)	Dob(u_1) girl(u_2) bit(u_1, u_2)	Dob(u_1) girl(u_2) bit(u_1, u_2)

This representation shows also that we need to represent how the discourse referents in the second and third column are connected to the referents in the first one. u_1 and u_2 in the first have been introduced by the same assertion as u_1 and u_2 in the second one, whereas u_1 and u_2 in the third one have been introduced together with u_3 and u_4. Hence, in a full representation we have to add functions $l_{i,H} : \mathcal{U}_{D_{i,H}} \longrightarrow \mathcal{U}_{D_{i,\{i\}}}$ to each column.

We can show in general that the DRSes $D_{a,H}$, which are internal representations of agent a for the referents and conditions which are common for the group H, are identical for all $a, b \in H$. If Bob meets Chris, then he can apply the uniqueness condition which is connected to the definite *the girl* to his DRS in the second column. He finds there u_2 as the only referent which represents a girl. As Chris will have the same representation for the common DRS, they both will interpret the description as relating to a subject which is *chained* to Melanie. But if Bob meets Debra, his use of *the girl* will single out the referent u_2 in the third column, which is chained to the girl Stephanie. The identity of the $D_{a,H}$'s explains why the group can co–ordinate their interpretation of referentially used definites. Furthermore, we can prove the following:

1. The set of common DRSes for a group H has maximal elements.
2. The maximal common DRSes for a group H are identical up to substitutions of variables.
3. The representing DRS for a group H is a maximal common DRS for this group.

This shows in which sense the DRSes $D_{a,H}$ are representations of the common ground: They are maximal common DRSes.

Part II
The Mathematics of the Theory

In this part we provide the formal description of our theory. We develop it in four sections. First we introduce the basic definitions relating to DRSes. In the second and third section of this part we introduce the two dialogue fragments as multi–agent systems. The first one describes the dialogue fragment with referential chains where the local states of the participants contain information about observed objects and actions they have been involved in. We denote the resulting class of games by $\mathcal{G}$. The second fragment has local states where the participants have explicit representations for the common information. Here, the set of resulting games is denoted by $\mathcal{G}^+$.

The following structure underlies the presentation of both fragments:

Actions		Games		Global States		Possibilities		Common Grounds
$\mathcal{A}(\mathcal{G})$	$\longleftarrow$	$\mathcal{G}$	$\longrightarrow$	$\mathcal{S}(\mathcal{G})$	$\longrightarrow$	$\mathcal{W}(\mathcal{G})$	$\longrightarrow$	$CG(\mathcal{G})$

I.e., in both cases we define a set $\mathcal{G}$ of games which describe all possible dialogues in the fragment. In the terminology of (Fagin e.al., 1995) such a set is called a set of *Runs* and is denoted by $\mathcal{R}$. But this terminology seems to be less well suited if we want to describe possible ways how a dialogue can develop. Every dialogue $G \in \mathcal{G}$ has the form

$$\langle s_0, \mathtt{act}_0, \ldots, \mathtt{act}_{n-1}, s_n \rangle ,$$

where the s_is denote global states, and the $\mathtt{act}_i$s dialogue acts. $\mathcal{A}(\mathcal{G})$ will denote the set of possible sequences of *actions*, $\mathcal{S}(\mathcal{G})$ the set of possible global states which actually can occur in a dialogue of $\mathcal{G}$. $\mathcal{W}(\mathcal{G})$ denotes the system where we include in the global states of participants additionally their *knowledge* about the global situation, and therefore, about each other. This will be done by use of the standard definitions for knowledge in multi–agent systems. This allows us to describe the common ground for each possibility as a special information state. The function from $\mathcal{S}(\mathcal{G})$ to $\mathcal{W}(\mathcal{G})$ will be bijective. The others are only surjective.

In the last section of this part we introduce *common DRSes*. They serve us as the central structure which allows us to relate the two defined dialogue fragments $\mathcal{G}$ and $\mathcal{G}^+$ to each other. There we will also provide the central theorems.

5 Basic Definitions

We provide the definitions relating to DRSes, i.e. the definitions of *DRSes*, *embeddings* of DRSes, *sub*–DRSes, *merge* of DRSes, and a relation between DRSes which we call *Dekker's support*.

Let $\mathcal{L}$ be a language of predicate logic, and $\mathfrak{M}$ a class of models for $\mathcal{L}$. We assume that $\mathcal{L}$ contains all the predicates the dialogue participants can use to talk about an outer situation.

If φ is a $\mathcal{L}$–formula, then we denote by $FV(\varphi)$ the *free variables* of φ, and by $BV(\varphi)$ the *bound* variables. We assume that there are two disjoint sets of variables $\mathcal{U} := \{u_i \mid i \in \mathbf{N}\}$ and $\mathcal{V}$ such that for all φ we have $FV(\varphi) \subset \mathcal{U}$ and $BV(\varphi) \subset \mathcal{V}$.

For $\mathfrak{A} \in \mathfrak{M}$ we denote by $|\mathfrak{A}|$ the universe of $\mathfrak{A}$. Let $|\mathfrak{M}| := \bigcup_{\mathfrak{A}\in\mathfrak{M}} |\mathfrak{A}|$.

Definition 5.1 (DRS) *A* proper DRS D *is a pair* $\langle \mathcal{U}_D, \mathrm{Con}_D \rangle$ *such that:*

- $\mathcal{U}_D$ *is a finite subset of* $\mathcal{U}$.
- $\mathrm{Con}_D = \{\varphi_1, \ldots, \varphi_m\}$, $m \in \mathbf{N}$, *is a finite set of* $\mathcal{L}$*–formulas such that for all* $\varphi \in \mathrm{Con}_D$ $FV(\varphi) \subseteq \mathcal{U}_D$.

If

- $\mathcal{U}_D = \{u_1, \ldots, u_n\}$ *for some* $n \in \mathbf{N}$,

then we call the DRS D regular.

Let $\mathfrak{A} \in \mathfrak{M}$. *For an* assignment *function* $f : \mathcal{U}_D \longrightarrow |\mathfrak{A}|$ *we define*

$$(\mathfrak{A}, f) \models D \textit{ iff } (\mathfrak{A}, f) \models \varphi \quad \textit{for all } \varphi \in \mathrm{Con}_D.$$

This allows us to define the meaning $[D]$ *of a DRS:*

$$[D] := \{(\mathfrak{A}, f) \mid \mathfrak{A} \in \mathfrak{M}, f : \mathcal{U}_D \longrightarrow |\mathfrak{A}|, (\mathfrak{A}, f) \models D\}.$$

Hence, the DRSes in our fragment have a very simple form, especially, we don't have subordination of DRSes, or modal contexts.

Definition 5.2 (Sub–DRS, Embeddings) *Let D and D' be proper DRS. Then let*

$$D \subseteq D' \text{ iff } \mathcal{U}_D \subseteq \mathcal{U}_{D'} \text{ and } \mathrm{Con}_D \subseteq \mathrm{Con}_{D'}.$$

For $f : \mathcal{U}_D \longrightarrow \mathcal{U}$ we define $f(D)$ by

$$\begin{array}{rcl} \mathcal{U}_{f(D)} & := & f[\mathcal{U}_D], \\ \mathrm{Con}_{f(D)} & := & \mathrm{Con}_D /_s f, \end{array}$$

where we define the substitution operation $/_s f$ as follows:

- *$\varphi /_s f$ is the result of substituting all free variables u in φ which are in the domain of f by their values $f(u)$.*
- *$m /_s f := \{m' /_s f \mid m' \in m\}$ for sets m.*

We introduce two merge operations $D_1 \oplus D_2$ and $D_1 \oplus_b D_2$. The first operation introduces for each discourse referent in D_2 a new referent into $\mathcal{U}_{D_1}$. $\oplus_b$ provides for the cases where some referents in D_2 should be *bound* to referents in D_1.

Definition 5.3 (Merge of DRS) *If D_1 and D_2 are two DRS whith $\mathcal{U}_{D_i} = \{u_1, \dots, u_{n_i}\}$, and $\mathrm{Con}_{D_i} = \{\varphi^i_1, \dots, \varphi^i_{m_i}\}$, then we define the* deterministic merge *$D_1 \oplus D_2$ of D_1 and D_2 by*

$$\begin{array}{rcl} \mathcal{U}_{D_1 \oplus D_2} & := & \{u_1, \dots, u_{n_1+n_2}\}, \\ \mathrm{Con}_{D_1 \oplus D_2} & := & \mathrm{Con}_{D_1} \cup \{\varphi^2_j /_s f \mid 1 \leq j \leq m_2\}, \end{array}$$

where $f : \mathcal{U}_{D_2} \longrightarrow \mathcal{U}$

$$f(u_j) := u_{n_1+j} \quad \text{for } j = 1, \dots, n_2.$$

We denote the class of all proper DRS by DRS.

If b is a partial function from $\mathcal{U}_{D_2}$ into $\mathcal{U}_{D_1}$, then we define the merge relative *b, $D_1 \oplus_b D_2$, in the following way: Let $\{v_1, \dots, v_m\}$ be the (order preserving) enumeration of $\mathcal{U}_{D_2} \setminus \mathrm{dom}\, b$, then*

$$\begin{array}{rcl} \mathcal{U}_{D_1 \oplus_b D_2} & := & \{u_1, \dots, u_{n_1+m}\}, \\ \mathrm{Con}_{D_1 \oplus_b D_2} & := & \mathrm{Con}_{D_1} \cup \{\varphi^2_j /_s f \mid 1 \leq j \leq m_2\}, \end{array}$$

where $f : \mathcal{U}_{D_2} \longrightarrow \mathcal{U}$

$$f(u_j) := \left\{ \begin{array}{ll} u_{n_1+i} & \text{for } u_j = v_i \\ b(u_j) & \text{for } u_j \in \mathrm{dom}\, b \end{array} \right. .$$

It follows that $D \oplus_b D' = D \oplus D'$ for $\mathrm{dom}\, b = \emptyset$.

Definition 5.4 (Dekker's Support) *Let D and D' be proper DRS. Let l be a function from $\mathcal{U}_{D'}$ to $\mathcal{U}_D$. Then D* supports *D' relative l iff $l(D') \subseteq D$.*

We follow (Dekker, 1997) and write $D \trianglelefteq_l D'$ if the relation holds. Originally the relation $\trianglelefteq_l$ was defined between *information aggregates*, i.e. sets of world–assignment pairs. If we assume that the information aggregates are just *meanings* of DRSes, then Dekker's original definition reads for functions $l : \mathcal{U}_{D'} \longrightarrow \mathcal{U}_D$ as follows (Dekker, 1997, p.30):

$$[D] \trianglelefteq_l [D'] \ \textit{iff} \ \forall(\mathfrak{A}, f) \in [D] \, \exists(\mathfrak{A}', f') \in [D'] \, \mathfrak{A} = \mathfrak{A}' \; \& \; f' = f \circ l.$$

It is easy to see that $D \trianglelefteq_l D' \Rightarrow [D] \trianglelefteq_l [D']$. The other direction $\Leftarrow$ does not hold in general because $D \trianglelefteq_l D'$ is a *syntactic* relation, whereas $[D] \trianglelefteq_l [D']$ is semantic. The distinction between DRSes is more fine-grained than that between information aggregates.

6 A Dialogue Fragment with Referential Chains

In this section we introduce the formal description of the fragment already discussed in Section 2. We will proceed in the following steps:

1. We define the set $\mathcal{G}$ of possible dialogues in our fragments as a set of games. We follow (Fagin e.al., 1995) and their definition of multi–agent systems. Hence we have to define

   (a) The set of possible joined actions ACT.

   (b) The set of possible global states $\mathcal{S}$.

   (c) The transition operation $\tau : \mathrm{ACT} \times \mathcal{S} \longrightarrow \mathcal{S}$

   (d) The Protocol P, which specifies which dialogue acts can be performed in which situation.

   (e) A set $\mathcal{S}_0$ of initial dialogue states.

   We first define a fragment where we allow only for *assertions* as dialogue acts. Then we add in a second step *observations*. We separated them because only assertions are in the focus of our interest, and observations are introduced just to be able to start with global states with empty representations.

2. Then we provide the definitions for:

   (a) The set $\mathcal{S}(\mathcal{G})$ of global states which can actually occur in the fragment characterised by $\mathcal{G}$.

   (b) The set of possibilities $\mathcal{W}(\mathcal{G})$, where we represent the knowledge of participants about the dialogue situation, and about each other.

   (c) The set $CG(\mathcal{G})$ of common grounds.

We prove some very technical lemmas which we need later on, Lemma 6.7, 6.9 and 6.10.

Definition 6.1 (Dialogue Acts, Extended DRS) *Let* DP *denote a finite set of possible* discourse participants. *For $A, i \in$* DP *we introduce the following sets of* local *acts:*

$$\mathrm{ACT}_A^{\mathtt{send}} := \{\mathtt{send}(A,H,D,l) \mid A \in \mathrm{DP}, A \notin H \subseteq \mathrm{DP}, D \in \mathrm{DRS}, l : \mathcal{U}_D \longrightarrow \mathcal{U}, l \textit{ injective}\},$$

and

$$\mathrm{ACT}_B^{\mathtt{get}} := \{\mathtt{get}(A,H,D) \mid A \in \mathrm{DP}, A \notin H \subseteq \mathrm{DP}, D \in \mathrm{DRS}, B \in H\}.$$

The set of all local *acts for* $i \in \mathrm{DP}$ *is defined by* $\mathrm{ACT}_i = \mathrm{ACT}_i^{\mathtt{send}} \cup \mathrm{ACT}_i^{\mathtt{get}} \cup \{\bot\}$.

An extended DRS D *is a triple* $\langle \mathcal{U}_D, \mathrm{Con}_D, Act_D\rangle$ *such that* $\langle \mathcal{U}, \mathrm{Con}_D\rangle$*is a proper DRS, and* Act_D *is a finite sequence of local acts for some participant* $i \in \mathrm{DP}$. *Let* DRS^e *denote the set of all extended DRSes.*

A joined *act is a sequence* $(\mathtt{act}_i)_{i\in\mathrm{DP}}$ *such that there is an* $A \in \mathrm{DP}$, $H \subseteq \mathrm{DP}$, $D \in \mathrm{DRS}$, *and* $l : \mathcal{U}_D \longrightarrow \mathcal{U}$ *with*

1. $\mathtt{act}_A = \mathtt{send}(A,H,D,l)$, *hence* $A \notin H$,
2. $\mathtt{act}_i = \mathtt{get}(A,H,D)$ *for* $i \in H$,
3. $\mathtt{act}_i = \bot$ *for* $i \in \mathrm{DP} \setminus (H \cup \{A\})$.

We denote the set of joined acts by ACT.

Definition 6.2 (Global States and Chain Relation) *For a fixed enumeration* $\{A_1,\ldots,A_n\}$ *of DP we denote by*

$$\mathcal{S} := \{\langle \mathfrak{A}, \rightarrow, D_{A_1},\ldots,D_{A_n}\rangle \mid \mathfrak{A} \in \mathfrak{M}, D_{A_i} \in \mathrm{DRS}^e\}$$

the set of global states. *We abbreviate* D_{A_i} *by* D_i, *and for* $H \subseteq \mathrm{DP}$ *we identify* H *with* $\{i \in \mathbf{N} \mid A_i \in H\}$. $\rightarrow$ *is a relation*

$$\rightarrow \subset (\mathrm{DP} \times \mathcal{U}) \times (\mathrm{DP} \times \mathcal{U}).$$

For $\langle\langle i,u\rangle,\langle j,v\rangle\rangle \in \rightarrow$, $i,j \in \mathrm{DP}$, $u,v \in \mathcal{U}$, *we will write* $u^i \rightarrow v^j$.

Definition 6.3 (Transition Operation) *We define a* transition function $\tau : \mathrm{ACT} \times \mathcal{S} \longrightarrow \mathcal{S}$: *Let* $\mathtt{act} = (\mathtt{act}_i)_{i\in\mathrm{DP}}$ *be a joined act and* $A \in \mathrm{DP}$ *with* $\mathtt{act}_A = \mathtt{send}(A,H,D,l)$. *Let* $s = \langle \mathfrak{A}, \rightarrow, D_1,\ldots,D_n\rangle$ *be a global state.*

Then, we define $\tau(\mathtt{act}, s) = \langle \mathfrak{A}', D_1',\ldots,D_n'\rangle$ *by*

1. $\mathfrak{A} = \mathfrak{A}'$
2. *For* $i \notin H$ *and* $i \neq A$: $D_i = D_i'$.
3. *For* $i \in H$ *we set*
   - $\langle \mathcal{U}_{D_i'}, \mathrm{Con}_{D_i'}\rangle = \langle \mathcal{U}_{D_i}, \mathrm{Con}_{D_i}\rangle \oplus D$,
   - $Act_{D_i'} = Act_{D_i} {}^\wedge \langle \mathtt{act}_i\rangle$.
4. *For* A *we get*
   - $\langle \mathcal{U}_{D_A'}, \mathrm{Con}_{D_A'}\rangle = \langle \mathcal{U}_{D_A}, \mathrm{Con}_{D_A}\rangle$,
   - $Act_{D_A'} = Act_{D_A} {}^\wedge \langle \mathtt{act}_A\rangle$.

5. *For* $i \in H$, $\mathcal{U}_{D_i} = \{u_1, \ldots, u_{n_i}\}$, *and* $l(u_k) = u_j$ *for* $u_k \in \mathcal{U}_D$ *and* $u_j \in \mathcal{U}_{D_A}$ *we have to add the following pairs to* $\rightarrow_s$ *in order to get* $\rightarrow_{\tau(\mathtt{act},s)}$*:*

$$u_j^A \rightarrow u_{n_i+k}^i.$$

We call the discourse referents in $\mathcal{U}_{D_i'} \setminus \mathcal{U}_{D_i}$ introduced *by* $\mathtt{act}_i$*, and we write* $D_i' = \mathtt{act}_i(D_i)$*.*

Definition 6.4 (Protocols and Dekker's Condition)
Let $s = \langle \mathfrak{A}, \rightarrow, D_1, \ldots, D_n \rangle$ *be a global state. Then we define the* protocol P *by*

$$P(s) = \{(\mathtt{act}_i)_{i \in \mathrm{DP}} \in \mathrm{ACT} \mid \mathtt{act}_A = \mathtt{send}(A, H, D, l) \Rightarrow D_A \trianglelefteq_l D\},$$

i.e. dialogue participant A *is* licensed *to make an utterance with content* D *iff the information contained by* D *is supported by his own information* D_A *relative to* l*. We call this condition* Dekker's Condition.

Definition 6.5 (Observations) *We extend the DRSes by a fourth component for the* observed objects*, i.e. we add a partial function* Obs_D *from* $\mathcal{U}_D$ *to* $|\mathfrak{M}|$*. Then, we add a new set of local acts:*

$$\mathrm{ACT}_i^{\mathtt{observe}} := \{\mathtt{observe}(H, D, l) \mid H \subseteq \mathrm{DP}, i \in H, D \in \mathrm{DRS}, l : \mathcal{U}_D \longrightarrow |\mathfrak{M}|\}$$

and add to the set of joined acts ACT *the set* $\mathrm{ACT}^{\mathtt{observe}}$ *of all* $(\mathtt{act}_i)_{i \in \mathrm{DP}}$ *such that there are* H*,* D *and* l *with*

- $\mathtt{act}_i = \mathtt{observe}(H, D, l)$ *iff* $i \in H$*,*
- $\mathtt{act}_i = \bot$ *iff* $i \notin H$*.*

For global states $s = \langle \mathfrak{A}, \rightarrow, D_1, \ldots, D_n \rangle$ *with chain relation, we add to the set* $P(s)$ *the actions* $\mathtt{act} = (\mathtt{act}_i)_{i \in \mathrm{DP}} \in \mathrm{ACT}^{\mathtt{observe}}$ *such that*

$$\mathtt{act}_i = \mathtt{observe}(H, D, l) \Rightarrow (\mathfrak{A}, l) \models D$$

The transition operation τ *is extended in the following way:* $\tau(\mathtt{act}, s) = s'$ *iff*

1. $\mathfrak{A} = \mathfrak{A}'$ *and* $D_i = D_i'$ *for* $i \notin H$*.*
2. *For* $i \in H$ *we have a partial function* b_i *with* $\mathrm{dom}\, b_i \subseteq \mathcal{U}_D$*, and*
   - $u \in \mathrm{dom}\, b_i$ *iff* $\exists v \in \mathcal{U}_{D_i}\, l(u) = \mathrm{Obs}_{D_i}(v)$*,*
   - $b_i(u) = v \Leftrightarrow l(u) = \mathrm{Obs}_{D_i}(v)$*.*

   Then we can get

   - $\langle \mathcal{U}_{D_i'}, \mathrm{Con}_{D_i'} \rangle = \langle \mathcal{U}_{D_i}, \mathrm{Con}_{D_i} \rangle \oplus_{b_i} D$*,*
   - $Act_{D_i'} = Act_{D_i} {}^\wedge \langle \mathtt{act}_i \rangle$*.*

3. *For* $i \in H$, $\mathcal{U}_{D_i} = \{u_1, \ldots, u_{n_i}\}$, *and* $l(v_k) = a$, *where* $a \in |\mathfrak{A}|$ *and* $\{v_1, \ldots, v_m\}$ *is the (order preserving) enumeration of* $\mathcal{U}_D \setminus \mathrm{dom}\, b$, *we have to add the following pairs to* $\rightarrow_s$ *in order to get* $\rightarrow_{s'}$:

$$a \rightarrow u^i_{n_i+k}.$$

The chain relation $\rightarrow$ *is now a subset of*

$$((\mathrm{DP} \times \mathcal{U}) \times (\mathrm{DP} \times \mathcal{U})) \cup (|\mathfrak{M}| \times (\mathrm{DP} \times \mathcal{U})).$$

Finally, we have to define $\mathrm{Obs}_{D'}$:

$$\mathrm{Obs}_{D'_i} := \mathrm{Obs}_{D_i} \cup \{\langle u^i_{n_i+k}, a\rangle \mid l(v_k) = a\}.$$

We write again $D'_i = \mathtt{act}_i(D_i)$. *Sometimes we will write* $v \in \mathrm{Obs}_D$ *instead of* $v \in \mathrm{dom}\mathrm{Obs}_D$.

Definition 6.6 (Dialogue Game) *A* dialogue game *is a sequence of the form* $G = \langle s_0, \mathtt{act}_0, s_1, \mathtt{act}_1, \ldots, s_n\rangle$, *where the* s_j *are global states with chain relations, the* $\mathtt{act}_j$ *are joined acts, and where for all* $j < n$

1. *if* $s_0 = \langle \mathfrak{A}, \rightarrow, D_1, \ldots, D_n\rangle$, *then the* D_i*s are* empty *DRSes,*
2. $\mathtt{act}_i \in P(s_j)$,
3. $s_{j+1} = \tau(\mathtt{act}_j, s_j)$.

We will denote the final state s_n *of* G *by* $G_f := \langle \mathfrak{A}_G, \rightarrow_G, G_i, \ldots, G_n\rangle$. *We denote the set of all games by* $\mathcal{G}$.

The following lemma characterises the different ways in witch two subjects u^i and v^j can be connected to an object x by the chain relation.

Lemma 6.7 *Let* $\rightarrow_r$ *denote the reflexive closure of the chain relation* $\rightarrow$. *Let* $x \rightarrow_r u^i$ *and* $x \rightarrow_r v^j$ *for* $i \neq j$. *Let* $G_f = \langle \mathfrak{A}, \rightarrow, D_1, \ldots, D_n\rangle$ *be a global state for a dialogue* $G \in \mathcal{G}$. *Then, one of the following cases holds:*

1. $u \in \mathrm{Obs}_{D_i}$ *and* $v \in \mathrm{Obs}_{D_j}$ $(\Rightarrow x \in |\mathfrak{A}|)$.
2. $u \notin \mathrm{Obs}_{D_i}$ *or* $v \notin \mathrm{Obs}_{D_j}$.
   (a) *If* $u \in \mathrm{Obs}_{D_i}$, *then* $u^i = x$ *and* v *was introduced into* D_j *by an act* $\mathtt{act}$ *where* $\mathtt{act}_j$ *is of the form* $\mathtt{get}(i, H, D)$ *for some* $H \subseteq \mathrm{DP}$ *with* $j \in H$, *and some DRS* D.
   (b) *If* $u \notin \mathrm{Obs}_{D_i}$ *and* $v \notin \mathrm{Obs}_{D_j}$, *then either*
      i. $x = u^i$ $(\Rightarrow x \neq v^j)$, *then* v *was introduced into* D_j *by an act* $\mathtt{act}$ *where* $\mathtt{act}_j$ *is of the form* $\mathtt{get}(i, H, D)$ *for some* $H \subseteq \mathrm{DP}$ *with* $j \in H$, *and some DRS* D.
      ii. $x \neq u^i$ *and* $x \neq v^j$, *then* u *and* v *were introduced into* $D_{i/j}$ *by an act* $\mathtt{act}$ *where* $\mathtt{act}_{i/j}$ *is of the form* $\mathtt{get}(A, H, D)$ *for some* $H \subseteq \mathrm{DP}$ *with* $i, j \in H$ $(\Rightarrow A \neq i, j)$, *and some DRS* D.

Proof: This lemma follows simply from the definition of the dialogues in $\mathcal{G}$ and the acts `observe` and `send`/`get`. □

Definition 6.8 (Global States for a Game $\mathcal{G}$) *Let $\mathcal{G}$ be given as in Definition 6.6. We denote by*

$$\mathcal{S}(\mathcal{G}) := \{s \in \mathcal{S} \mid \exists G \in \mathcal{G}\, G_f = s\}$$

the set of all possible final states for $\mathcal{G}$.

The following lemma collects some useful observations about the chain relation, discourse referents, and the way they have been introduced into the local DRSes by actions.

Lemma 6.9 *Let $s \in \mathcal{S}(\mathcal{G})$, $i \in \mathrm{DP}$. Let $\to_r$ denote the reflexive closure of $\to$. Assume that $s = G_f = s_n$ for some $G = \langle s_0, \mathtt{act}_1, \ldots, \mathtt{act}_n, s_n\rangle \in \mathcal{G}$. Then*

1. *If $u \in \mathcal{U}_{G_i}$, then there exists exactly one $1 \leq m \leq n$ such that u was introduced by $\mathtt{act}_m$. Moreover, if $u \in FV(\varphi)$, and $\mathtt{act}_m$ is not an observation, then all $v \in FV(\varphi)$ and φ itself are introduced by the same $\mathtt{act}_m$.*
2. *If $u^i \to v^j$, and if u^i was introduced by $\mathtt{act}_{m_1}$ and v^j by $\mathtt{act}_{m_2}$, then $m_1 < m_2$ and $\mathtt{act}_{m_2}$ is not an observation. It follows especially that $\to$ is irreflexive, i.e. $u^i \to v^j \Rightarrow v^j \not\to u^i$.*
3. *$x \to u^i$ and $y \to u^i$ then $x = y$.*
4. *If $x \to u^i$, and if u was introduced by $\mathtt{act}_m$, where $(\mathtt{act}_m)_i$ is of the form $\mathtt{get}(A, H, D)$, then x is of the form v^A.*

Proof: We denote by $lh(G) := n$ for $G = \langle s_0, \mathtt{act}_1, \ldots, \mathtt{act}_n, s_n\rangle$ the *length* of G. Let $s_m = \langle \mathfrak{A}, \to^m, D_i^m\rangle_{i\in\mathrm{DP}}$. We prove the lemma by induction over $lh(G)$.

1. For $lh(G) = 0$ is nothing to show. Assume that $lh(G) = n+1$. If u is introduced by $\mathtt{act}_{n+1}$, then it can't be an element of $\mathcal{U}_{D_i^n}$. Hence, it is only introduced by $\mathtt{act}_{n+1}$. For u not introduced by $\mathtt{act}_{n+1}$ it follows that $u \in \mathcal{U}_{D_i^n}$, therefore, by I.H., it was introduced by exactly one $\mathtt{act}_m$, $m < n+1$. Let $u \in FV(\varphi)$, and assume that u was introduced by an $\mathtt{act}_m$ which was not an observation. Hence, it follows by induction that $u \notin \mathrm{dom}\,\mathrm{Obs}_{D_i^{n+1}}$. If φ would have been introduced by an observation, then $FV(\varphi)$ would be a subset of the domain of $\mathrm{Obs}_{D_i^{n+1}}$ but this contradicts the assumption. Hence, φ was not introduced by an observation. Suppose it was introduced by an $\mathtt{act}_{m'}$ with $m' \neq m$. Then $(\mathtt{act}_{m'})_i$ is of the form $\mathtt{get}(A, H, D)$, $i \neq A$. But then, all variables in $FV(\varphi)$ have been introduced in this step, which again contradicts the assumption. This shows, that φ and all variables in $FV(\varphi)$ are introduced with the same act as u.

2., 3. and 4. follow by induction over $lh(G)$.

□

Lemma 6.10 *Let $s \in \mathcal{S}(\mathcal{G})$, $i \in \mathrm{DP}$. Let $\to_r$ denote the reflexive closure of $\to$. Assume that $s = G_f = s_n$ for some $G = \langle s_0, \mathtt{act}_1, \ldots, \mathtt{act}_n, s_n\rangle \in \mathcal{G}$. Then*

1. *Let $u^A \to_r v_k^{i_k}$, $k \in \{0, 1\}$, $\varphi_0 \in \mathrm{Con}_{D_{i_0}}$, $v_0 \in FV(\varphi_0)$. Suppose that $v_0^{i_0}$ and $v_1^{i_1}$ have been introduced by the same $\mathtt{act}_m$, then there exist $\varphi \in \mathrm{Con}_{D_A}$, $\varphi_1 \in \mathrm{Con}_{D_{i_1}}$ and unique bijective $f_k : FV(\varphi_k) \longrightarrow FV(\varphi)$ such that:*

(a) $\forall v \in FV(\varphi_k)\; f_k(v)^A \rightarrow_r v_k^{i_k}$,

(b) $\varphi = \varphi_k /_s f_k$,

(c) If $g : FV(\varphi_0) \longrightarrow FV(\varphi_1)$, $g(v) := v'$ iff $f_0(v) = f_1(v')$, then $\varphi_1 = \varphi_0 /_s g$.

2. *If a joined act contains a local act of the form* $\mathtt{get}(A, D, H)$, *then there exist for* $k \in \{A\} \cup H$ *unique injective functions* $l_k : \mathcal{U}_D \longrightarrow \mathcal{U}_{D_k}$ *such that* $\forall k \in H\, \forall u \in \mathcal{U} : l_k(u)$ *was introduced by this joined act into* D_k *and* $l_A(u) \rightarrow l_k(u)$. *It holds* $l_k(D) \subseteq D_k$.

3. *Let* $\mathtt{act}$ *be a joined act which contains a local act* $\mathtt{act}_i = \mathtt{get}(A, D, H)$, $i \in H$. *Let* $(l_k)_{k \in H \cup \{A\}}$ *be given as in 2. Let* D' *be a DRS, and let* $f_i : \mathcal{U}_{D'} \longrightarrow \mathcal{U}_{D_i}$ *be injective with* $f_i(D') \subseteq l_i(D)$. *Then there exists a unique family* $(f_k)_{k \in H \cup \{A\}}$ *such that for all* $k \in H \cup \{A\}$ *(1)* $f_k : \mathcal{U}_{D'} \longrightarrow \mathcal{U}_{D_k}$, *(2)* $f_k(D') \subseteq l_k(D)$, *and (3)* $\forall u \in \mathcal{U}_{D'}\; l_i^{-1}(f_i(u)) = l_k^{-1}(f_k(u))$. *It follows that* $\forall u \in \mathcal{U}_{D'}\; \forall k \in H\; f_A(u)^A \rightarrow f_k(u)^k$.

Proof: 1. As both, v_0 and v_1, are introduced by the same $\mathtt{act}_m$, it follows that $A \neq i_k$, and that $(\mathtt{act}_m)_k$ is of the form $\mathtt{get}(A, H, D)$ with $i_k \in H$, $k = 0, 1$, especially, $\mathtt{act}_m$ can't be an observation. But then there is an $\varphi' \in \mathrm{Con}_D$ such that v_0 was introduced by a $v \in FV(\varphi')$ into $\mathcal{U}_{D^m_{i_0}}$. $(\mathtt{act}_m)_A$ must have been of the form $\mathtt{send}(A, H, D, l)$. It is clear that $\varphi := \varphi' /_s l$ is the desired formula, and that we can get f_k from the definition of $\tau(\mathtt{act}_m, s_{m-1})$, and that the f_ks are unique. 2. is a consequence of 1. The family of function $(f_k)_{k \in H \cup \{A\}}$ in 3. can be defined with the functions introduced in 1. and 2. □

There is a standard way to define the knowledge of an agent in a multi–agent system. Basically, this is done in a possible worlds framework, i.e. the beliefs of an agent are identified with a set of global states. This set is defined as the set of states which he can not distinguish from the actual one, i.e. where his local state is identical with his actual local state.

Definition 6.11 *Let* $\mathcal{G}$ *be given as in Definition 6.6. We define an equivalence relation* $\sim_i$ *for* $i \in \mathrm{DP}$ *on* $\mathcal{G} \times \mathcal{G}$

$$G \sim_i G' \text{ iff } G_i = G'_i.$$

$G \sim_i G'$ *means that* G *and* G' *are* indistinguishable *for agent* i. G_i *and* G'_i *denote here the local states of* i *in the final situations of* G *and* G', *see Definition 6.6. We set*

$$[D]_i := \{G \in \mathcal{G} \mid G_i = D\}$$

For $s \in \mathcal{S}(\mathcal{G})$, $s = \langle \mathfrak{A}, \rightarrow, D_1, \ldots, D_n \rangle$, *we set*

$$[s]_i := \{s' \in \mathcal{S}(\mathcal{G}) \mid \exists G \in [D_i]_i\; s' = G_f\}.$$

There are different ways to explicitly represent the knowledge of agents. One way is do define *Kripke structures* $\mathcal{K} = \langle \mathcal{S}(\mathcal{G}), K_1, \ldots, K_n \rangle$, where the K_i's represent accessibility relations for the agents $i = 1, \ldots, n$. Here, a global state s' is accessible in state s for agent i iff the local states of i are indiscernible, i.e. iff $s' \in [s]_i$. A specific dialogue situation can then be identified with the pair $\langle s, \mathcal{K} \rangle$, or with s itself because $\mathcal{K}$ is fixed for $\mathcal{G}$.

Another way to represent the information of agents in a multi–agent system is provided by the so–called *possibility approach* developed in the paper (Gerbrandy & Groeneveld, 1997). It directly adds the information states of agents to the global states. Both ways lead to equivalent representations[1]. We don't want to go into the technical details of the definition of possibilities. It is developed in a set theory with a special Anti–Foundation Axiom introduced by *Peter Aczel* (Aczel, 1988). In order to clearly separate the structures with and without explicit representation of knowledge we use the concept of possibilities in all subsequent sections. But it is always possible to read them as the related representations based on Kripke structures.

Definition 6.12 (Possibilities) *For* $s \in \mathcal{S}(\mathcal{G})$, $s = \langle \mathfrak{A}, \rightarrow, D_1, \ldots, D_n \rangle$, *and* $\sigma \subseteq \mathcal{S}(\mathcal{G})$ *we define*

- $I(s) = \langle \mathfrak{A}, \rightarrow, \langle D_1, I([s]_1) \rangle, \ldots, \langle D_n, I([s]_n) \rangle \rangle$,
- $I(\sigma) = \{I(s) \mid s \in \sigma\}$.

We call $I(s)$ *a* possibility, *and* $I(\sigma)$ *an* information state. *We denote the set of all possibilities for* $\mathcal{S}(\mathcal{G})$ *by*

$$\mathcal{W}(\mathcal{G}) = \{I(s) \mid s \in \mathcal{S}(\mathcal{G})\}.$$

We write I_i^w *for the information states of agent* i *in a given possibility* $w \in \mathcal{W}(\mathcal{G})$.

If one prefers Kripke structures, then $I(s)$ is just $\langle s, \mathcal{K} \rangle$, or s itself. $I([s]_i)$ can be read as $[s]_i$.

Definition 6.13 (Common Ground) *We get the* common ground $CG_H(w)$ *for a group* H *of agents and a possibility* $w = \langle \mathfrak{A}, \rightarrow, \langle D_1, I_1 \rangle, \ldots, \langle D_n, I_n \rangle \rangle \in \mathcal{W}$ *as the smallest transitive superset of all* I_i *with* $i \in H$, *i.e. the smallest set* σ *with*

- $\forall i \in H\, I_i^w \subseteq \sigma$,
- $v \in \sigma \Rightarrow \forall i \in H\, I_i^v \subseteq \sigma$.

We denote the set of all common grounds for $\mathcal{G}$ *by* $CG(\mathcal{G})$.

For Kripke structures these definitions read as follows: A common ground for s and $\mathcal{K}$ is the smallest set σ closed under

- $\forall i \in H\, [s]_i \subseteq \sigma$,
- $s' \in \sigma \Rightarrow \forall i \in H\, [s']_i \subseteq \sigma$.

[1] See (Gerbrandy & Groeneveld, 1997), and especially (Gerbrandy, 1998, Prop. 3.7).

7 A Fragment with Representations for Common DRSes

In this section we introduce the dialogue fragment where the participants maintain explicit representations for the information and discourse referents they have in common with others. We describe it by a new set of games $\mathcal{G}^+$. The underlying structure for the following definitions is the same as for Section 6. We don't include the chain relation in the global states because we are only interested in the way how the participants maintain the explicit representations. But, of course, it would be no problem to do so.

Definition 7.1 *Let $\mathcal{S}^+$ be the set of all $\langle \mathfrak{A}, f_1, \ldots, f_n \rangle$ such that*

1. $\mathfrak{A} \in \mathfrak{M}$,
2. $fkt(f_i), \mathrm{dom}\, f_i = \{H \in \mathcal{P}(\mathrm{DP}) \mid i \in H\}$,
3. $f_i(H) = \langle l_{i,H}, D_{i,H} \rangle$ *with* $D_{i,H} \in \mathrm{DRS}$ *and* $D_{i,\{i\}} \trianglelefteq_{l_{i,H}} D_{i,H}$.

Let $\langle \mathfrak{A}, f_1, \ldots, f_n \rangle \in \mathcal{S}^+$ be given. $\mathcal{U}_{D_{i,H}} = \{u_1, \ldots, u_{n(H)}\}$. For the local acts $\mathtt{act}_i \in \mathrm{ACT}_i$ we define local updates. Let $\tau_i(\mathtt{act}_i, f_i)(H) := \langle l', D' \rangle$, $i \in H \subseteq G$, $\mathcal{U}_D = \{u_1, \ldots, u_m\}$, where $\langle l', D' \rangle$ is given by the following conditions:

- $\mathtt{act}_i = \mathtt{observe}(G, D, l)$ *with* $i \in G$ *and* $\{v_1, \ldots, v_m\}$ *the enumeration of* $\{u \in \mathcal{U}_D \mid l(u) \notin \mathrm{ranObs}_D\}$, *then*
  1. $D' = \mathtt{act}_i(D_{i,H})$, *see Definition 6.5, p. 21.*
  2. $l'(u_{n+k}) := l(v_k)$ *for* $1 \leq k \leq m$, *and for* $1 \leq k \leq n$ $l'(u_k) = l_{i,H}(u_k)$.
- $\mathtt{act}_i = \mathtt{send}(i, G, D, l)$ *with* $i \in G$, *then*
  1. $\langle l', D' \rangle = \langle l_{i,H}, D_{i,H} \rangle$ *for* $H = \{i\}$.
  2. $D' = D_{i,H} \oplus D$ *for* $H \neq \{i\}$
  3. $l'(u_{n+k}) := l(k)$ *for* $1 \leq k \leq m$, *and for* $1 \leq k \leq n$ $l'(u_k) = l_{i,H}(u_k)$.
- $\mathtt{act}_i = \mathtt{get}(i, G, D)$ *with* $i \in G$, *then*
  1. $D' = D_{i,H} \oplus D$,
  2. $l'(u_{n_H+k}) := u_{n_{\{i\}}+k}$ *for* $1 \leq k \leq m$, *and for* $1 \leq k \leq n_H$ $l'(u_k) = l_{i,H}(u_k)$.

Then, the global transition function τ for a joined act $(\mathtt{act}_i)_{i \in \mathrm{DP}} =: \mathtt{act}$ is given by:

$$\tau(\mathtt{act}, \langle \mathfrak{A}, f_1, \ldots, f_n \rangle) := \langle \mathfrak{A}, \tau_1(\mathtt{act}_1, f_1,), \ldots, \tau_n(\mathtt{act}_n, f_n) \rangle .$$

For $s = \langle \mathfrak{A}, f_1, \ldots, f_n \rangle$ we define the Protocol P *by*

$$\begin{aligned} P(s) \quad := \quad & \{(\mathtt{act}_i)_{i \in \mathrm{DP}} \in \mathrm{ACT} \mid \mathtt{act}_A = \mathtt{send}(A, G, D, l) \Rightarrow D_{A,\{A\}} \trianglelefteq_l D\} \cup \\ & \{(\mathtt{act}_i)_{i \in \mathrm{DP}} \in \mathrm{ACT} \mid \mathtt{act}_i = \mathtt{observe}(G, D, l) \Rightarrow (\mathfrak{A}, l) \models D\}. \end{aligned}$$

The games in $\mathcal{G}^+$ should again start in situations where the participants have no entries in their representing DRSes. Hence, the class of *initial dialogue situations* $\mathcal{S}_0^+$ is the set of all $\langle \mathfrak{A}, f_1, \ldots, f_n \rangle \in \mathcal{S}^+$ such that for all $H \subseteq \mathrm{DP}$, $1 \leq i \leq n$ $f_i(H)$ is empty. Therefore, we can identify the initial states for $\mathcal{G}$ and $\mathcal{G}^+$ and write $\mathcal{S}_0$ for $\mathcal{S}_0^+$.

We can use the formulations of the definitions for dialogue games, Definition 6.6, global states for a game, Definition 6.8, possibilities, Definition 6.12, and the common ground, Definition 6.13 to define the related structures for $\mathcal{S}^+$. For the definition of possibilities we set

$$[s]_i := \{s' \in \mathcal{S}(\mathcal{G}^+) \mid f_i = f_i'\}.$$

The following theorem claims that all members of a group H maintain the same representations for the common DRS of this group. The proof is quite simple. We nevertheless call it a theorem due to it's empirical importance. If this result would not hold, then it would be difficult to explain how the discourse participant can successfully coordinate their dialogue acts, especially the referential anaphoric use of definite descriptions.

Theorem 7.2 *Let $s = \langle \mathfrak{A}, f_1, \ldots, f_n \rangle \in \mathcal{S}(\mathcal{G}^+)$, $f_i(H) = \langle l_{i,H}, D_{i,H} \rangle$. Then*

$$\forall H \subseteq \mathrm{DP}\, \forall i, j \in H\, D_{i,H} = D_{j,H}$$

It follows by induction over the length $lh(G)$ of games $G \in \mathcal{G}^+$. This theorem allows us to write D_H for $D_{i,H}$, $i \in H$.

In the next section we will investigate the relation between the common ground in $\mathcal{G}$ and the common representations in $\mathcal{G}^+$. The central concept will be that of a common DRS. For this investigation we need to compare games in $\mathcal{G}$ and $\mathcal{G}^+$, hence we need to be able to relate them to one another. This is done by use of the sequences of actions which are part of the dialogues in $\mathcal{G}$ and $\mathcal{G}^+$.

Definition 7.3 *If $G = \langle s_0, \mathtt{act}_0, \ldots, s_{n+1} \rangle$ is an element of $\mathcal{G}$ or $\mathcal{G}^+$, then we denote by $a(G) = \langle \mathtt{act}_0, \ldots, \mathtt{act}_n \rangle$ the related sequence of actions.*

Let $\mathcal{A}(\mathcal{G}^{(+)})$ be the set of all finite sequences of actions which can occur in $\mathcal{G}$ or $\mathcal{G}^+$, i.e.

$$\mathcal{A}(\mathcal{G}^{(+)}) := \{a(G) = \langle \mathtt{act}_0, \ldots, \mathtt{act}_n \rangle \mid n \in \mathbf{N}, G \in \mathcal{G}^{(+)}\}.$$

By induction we can show that $\mathcal{A}(\mathcal{G}) = \mathcal{A}(\mathcal{G}^+)$. Hence, we can write just $\mathcal{A}$ for $\mathcal{A}(\mathcal{G}^{(+)})$. This allows us to define for both games $\mathcal{G}$ and $\mathcal{G}^+$ and their initial states $s \in \mathcal{S}_0$ the sets

$$\mathcal{A}(s) := \{a(G) \mid G \in \mathcal{G}^{(+)}, G = \langle s_0, \mathtt{act}_0, \ldots, \mathtt{act}_n, s_{n+1} \rangle \;\&\; s_0 = s\}.$$

The following definition gives names to the maps in the following schema:

$$\mathcal{A} \quad \xleftarrow{a} \quad \mathcal{G}^{(+)} \quad \xrightarrow{S} \quad \mathcal{S}(\mathcal{G}^{(+)}) \quad \xrightarrow{I} \quad \mathcal{W}(\mathcal{G}^{(+)}) \quad \xrightarrow{CG} \quad CG(\mathcal{G}^{(+)})$$

The function γ maps a possible sequence of actions for some initial state to the possibility correlated to it's resulting final state.

Definition 7.4 • $S : \mathcal{G}^{(+)} \longrightarrow \mathcal{S}(\mathcal{G}^{(+)})$, $G \mapsto G_f$.

- $I^{(+)} : \mathcal{S}(\mathcal{G}^{(+)}) \longrightarrow \mathcal{W}(\mathcal{G}^{(+)})$, $s \mapsto I(s)$, *see Def. 6.12.*
- *For a given initial situation s we can define a function which provides for each sequence of actions in $\mathcal{A}(s)$ the resulting final possibility:* $\gamma_s^{(+)} : \mathcal{A}(s) \longrightarrow \mathcal{W}(\mathcal{G}^{(+)})$, $\gamma_s^{(+)}(a) = I(S(G(s,a)))$ *where $G(s,a)$ denotes the game in $\mathcal{G}^{(+)}$ which is uniquely determined by s and a. If s is given by context, then we write just $\gamma^{(+)}$.*
- *For $H \subseteq DP$ and $w \in \mathcal{W}(\mathcal{G}^{(+)})$ let $CG_H(w)$ denote the smallest set $\sigma \subseteq \mathcal{W}(\mathcal{G}^{(+)})$ such that for all $i \in H$ $I_i^w \subseteq \sigma$, and such that $\forall i \in H \forall v (v \in \sigma \Rightarrow I_i^v \subseteq \sigma)$.*

The following lemma states that in both games, $\mathcal{G}$ and $\mathcal{G}^+$, the participants build for the same sequence of dialogue acts the same DRSes which represent their total knowledge about the situation talked about.

Lemma 7.5 *Let $s \in \mathcal{S}_0$, $a \in \mathcal{A}(s)$, $i \in$ DP. Then*

$$D_i^{\gamma_s(a)} = D_{i,\{i\}}^{\gamma_s^+(a)}.$$

Proof: Follows by induction over the length of a. □

8 Common DRSes

We introduce the notion of a *common DRS*. The main result of this section, Theorem 8.11, shows that the explicit representations $D_H^{\gamma_s^+(a)}$ for a possibility $\gamma_s^+(a)$ in $\mathcal{W}(\mathcal{G}^+)$ is in fact *the maximal common DRS* for a group H in $\gamma_s(a)$ in $\mathcal{W}(\mathcal{G})$.

Definition 8.1 *Let $\rightarrow_r$ denote the* reflexive *closure of a given chain relation $\rightarrow$. Let $w = \langle \mathfrak{A}, \rightarrow, \langle D_1, I_1\rangle, \ldots, \langle D_m, I_m\rangle\rangle \in \mathcal{W}(\mathcal{G})$. Let $H \subseteq$ DP $= \{1, \ldots, m\}$.*

1. $u^i \sim_k v^j$ *iff* $\exists t \in \mathcal{U}_{D_k}\, t^k \rightarrow_r u^i \;\&\; t^k \rightarrow_r v^j$.
2. $u^i \sim_0 v^j$ *iff* $\exists t \in |\mathfrak{A}|\, t \rightarrow_r u^i \;\&\; t \rightarrow_r v^j$.
3. $u^i \sim_x v^j$ *iff* $\exists k \geq 0\; u^i \sim_k v^j$.
4. *For $D^i \subseteq D_i$ and $D^j \subseteq D_j$ let $D^i \sim_{ij} D^j$ iff*
   (a) $\exists f : \mathcal{U}_{D^i} \longrightarrow \mathcal{U}_{D^j}\, \forall u \in \mathrm{dom}\, f\, u^i \sim_x f(u)^j \;\&\; f(D^i) \subseteq D^j$, *and*
   (b) $\exists f : \mathcal{U}_{D^j} \longrightarrow \mathcal{U}_{D^i}\, \forall u \in \mathrm{dom}\, f\, u^j \sim_x f(u)^i \;\&\; f(D^j) \subseteq D^i$.

$u^i \sim_k v^j$ means that the subjects u^i and v^j are chained to the same subject in the local DRS of participant k. $u^i \sim_0 v^j$ means that they are chained to the same object in the described situation.

In the following definition $E_w(l, D, H)$ means that the DRS D is a *joined* DRS for the group H in world w relative to l. We motivated this definition in Section 3. $C_w(l, D, H)$ then means that D is common for H in w relative l, and the definition for E^{n+1} contains the conditions for the iteration of *joinedness*. E plays a similar role as the modal operator E for *everybody* in the usual definitions of *common knowledge*, see (Fagin e.al., 1995).

Definition 8.2 (Common DRS)
Let D be a DRS and $w = \langle \mathfrak{A}, \rightarrow, \langle D_1, I_1 \rangle, \ldots, \langle D_m, I_m \rangle \rangle \in \mathcal{W}(\mathcal{G})$. Let $H \subseteq \mathrm{DP}$, $\mathrm{DP} = \{1, \ldots, m\}$. Then

1. $E_w(l, D, H)$ *iff*
   (a) $l = l^{(w)} := (l_i^w)_{i \in H}$ *is a family of functions* $l_i : \mathcal{U}_D \longrightarrow \mathcal{U}_{D_i}$,
   (b) $D_i \trianglelefteq_{l_i} D$,
   (c) $\forall u_1, u_2 \in \mathcal{U}_D : u_1 \neq u_2 \Rightarrow \exists i \in H : l_i(u_1) \neq l_i(u_2)$,
   (d) $\forall i, j \in H \, l_i(D) \sim_{ij} l_j(D)$.
2. $E_w^1(l, D, H)$ *iff* $E_w(l, D, H)$,
3. $E_w^{n+1}(l, D, H)$ *iff* $E_w(l, D, H)$ & $\forall i \in H \, \forall v \in I_i \, \exists l^v = \left(l_j^v\right)_{j \in H}$:
   (a) $l_i^w = l_i^v$
   (b) $\forall u \in \mathcal{U}_D \, \forall j, j' \in H \, \forall k \geq 0 (l_j^w(u)^j \sim_k l_{j'}^w(u)^{j'} \Leftrightarrow l_j^v(u)^j \sim_k l_{j'}^v(u)^{j'})$
   (c) $E_v^n(l^v, D, H)$.
4. $C_w(l, D, H)$ *iff* $\forall n \in \mathbf{N} \, E_w^n(l, D, H)$.
5. $C_w(D, H)$ *iff* $\exists l = (l_i)_{i \in H}$ & $C_w(l, D, H)$.

If $C_w(D, H)$, then we call D common *for possibility w.*

The condition

$$\forall u \in \mathcal{U}_D \, \forall j, j' \in H \, \forall k \geq 0 \, (l_j^w(u)^j \sim_k l_{j'}^w(u)^{j'} \Leftrightarrow l_j^v(u)^j \sim_k l_{j'}^v(u)^{j'})$$

in the definition of $E^{n+1}(l, D, H)$ is necessary to prove the maximality of the representing DRSes $D_H^{w^+}$ in $\mathcal{G}^+$. The condition implies that for every referent in $D_H^{w^+}$ it is common information that it was introduced by an observation, or it is common information that it was introduced by the same dialogue participant.

It is useful to define a stronger conditions for joinedness. $E_w(l, D, H, k)$ says that all variables in the $l_i(D)$s are chained to subjects in the local DRS of k, $k > 0$. For $k = 0$ it says that they are chained to objects in the described situation.

Definition 8.3 *Let D be a DRS and $w = \langle \mathfrak{A}, \rightarrow, \langle D_1, I_1 \rangle, \ldots, \langle D_m, I_m \rangle \rangle \in \mathcal{W}(\mathcal{G})$. Let $H \subseteq \mathrm{DP}$, $\mathrm{DP} = \{1, \ldots, m\}$. Then*

1. $E_w(l, D, H, k)$ *iff*
   (a) $l = l^{(w)} := (l_i^w)_{i \in H}$ *is a family of functions* $l_i : \mathcal{U}_D \longrightarrow \mathcal{U}_{D_i}$ *and* $D_i \trianglelefteq_{l_i} D$,
   (b) $k \in \mathrm{DP}$,
   (c) $\forall u_1, u_2 \in \mathcal{U}_D : u_1 \neq u_2 \Rightarrow \exists i \in H : l_i(u_1) \neq l_i(u_2)$,
   (d) $\forall i, j \in H \exists f_i : \mathcal{U}_{l_i(D)} \longrightarrow \mathcal{U}_{D_k}$
      i. $\forall u \in \mathcal{U}_{l_i(D)} \, f_i(u)^k \rightarrow_r u^i$,
      ii. $D_k \trianglelefteq_{f_i} l_i(D)$,

iii. $\forall u \in \mathcal{U}_D\, f_i(l_i(u)) = f_j(l_j(u))$.

2. $E_w(l, D, H, 0)$ *iff*

   (a) $l = l^{(w)} := (l_i^w)_{i \in H}$ *is a family of functions* $l_i : \mathcal{U}_D \longrightarrow \mathcal{U}_{D_i}$ *and* $D_i \trianglelefteq_{l_i} D$,

   (b) $\forall u_1, u_2 \in \mathcal{U}_D : u_1 \neq u_2 \Rightarrow \exists i \in H : l_i(u_1) \neq l_i(u_2)$,

   (c) $\forall u \in \mathcal{U}_D \forall i, j \in H \operatorname{ran} l_{i/j} \subseteq \mathrm{Obs}_{D_{i/j}}\ \&\ \mathrm{Obs}_{D_i}(l_i(u)) = \mathrm{Obs}_{D_j}(l_j(u))$.

Lemma 8.4 *1. For* $i \neq j$ $u^i \sim_A v^j\ \&\ u^i \sim_B v^j \Rightarrow A = B$.

2. If $\varphi_i \in \mathrm{Con}_{D_i}$, $\varphi_j \in \mathrm{Con}_{D_j}$ *and*

- $\forall u \in FV(\varphi_i) \exists v \in FV(\varphi_j) \exists A_u\, u^i \sim_{A_u} v^j$ *and*
- $\forall v \in FV(\varphi_j) \exists u \in FV(\varphi_i) \exists A_v\, u^i \sim_{A_v} v^j$,

then all the A_u*s and* A_v*s are identical.*

Proof:

1. $i \neq j$ implies $A \neq i$ or $A \neq j$. Assume $A \neq i$. Then, there is a $r \in \mathcal{U}_{D_A}$ such that $r^A \to v^i$. If also $B \neq i$, then there exists a $s \in \mathcal{U}_{D_B}$ such that $s^B \to v^i$. Lemma 6.9 implies $r^A = s^B$. If $B = i$, then $u^i \to v^j$ by Lemma 6.7, and therefore $u^i = r^A$ by Lemma 6.9. Hence $A = B$.

2. With Lemma 6.9 it follows that φ_i and all $u \in FV(\varphi_i)$ have been introduced by the same $\mathtt{act}_{m_i}$. The same holds for j. Hence, if $i = j$, the proposition follows directly. Therefore, assume $i \neq j$. If $m_i = m_j$, then the claim follows with Lemma 6.10. Assume $m_i < m_j$. Hence $\mathtt{act}_{m_j}$ is not an observation, and $(\mathtt{act}_{m_j})_j$ is of the form $\mathtt{get}(A, G, D)$. It follows with Lemma 6.9 that $i = A$.

□

In the following proofs we very often find a situation where some action was performed for a group H, and we want to show that for an agent who does not belong to this group it might be replaced by a sequence of actions where only single persons where involved. Then, we will also find situations where we have two actions where the second depends on the first, and we want to show that for some agent it would be possible that this dependence does not exist. In these cases we can make use of the following remark.

Remark 8.5 (Replacement) *Let* $G \in \mathcal{G}$ *be given, let* a *be the sequence of actions which belongs to* G*. Let* s *be the initial situation, and* $\gamma_s(a)$ *the related final possibility.*

Assume that a joined act with $\mathrm{Obs}(H, D, l)$ *is part of the sequence* a*. If we replace it by a sequence of joined actions with local acts* $\mathrm{Obs}(\{i\}, D, l)$*, one for each participant* $i \in H$*, then the new final state will be indiscernible from* $\gamma_s(a)$ *for all* $j \in \mathrm{DP} \setminus H$*.*

In the following we write D_φ for the DRS $\langle FV(\varphi), \{\varphi\}\rangle$. We say that D *represents* φ iff there exists a $\varphi' \in \mathrm{Con}_D$, and an $f : FV(\varphi') \longrightarrow FV(\varphi)$ such that $\varphi' /_s f = \varphi$.

We want to show that every common DRS for a situation in game $\mathcal{G}$ can be embedded into the local representations in $\mathcal{G}^+$. We will reach this goal in Theorem 8.11. But before we can proof the theorem we have to show a number of preparatory lemmas.

Lemma 8.6 shows that we can uniquely decompose a joined DRS into the parts which are chained to the environment, or to the various discourse participants.

Lemma 8.6 *Let D be a DRS and $w = \langle \mathfrak{A}, \to, \langle D_1, I_1\rangle, \dots, \langle D_m, I_m\rangle\rangle \in \mathcal{W}(\mathcal{G})$. Let $H \subseteq \mathrm{DP}$, $\mathrm{DP} = \{1, \dots, m\}$, $|H| > 1$. Then the following three conditions are equivalent*

1. $E_w(l, D, H)$
2. *There is exactly one sequence $(D_k)_{k=0,\dots,m}$ such that*
   (a) $\biguplus_{k=0,\dots,m} D_k = D$
   (b) $\forall k = 0, \dots, m\ E_w(l, D_k, H, k)$
3. *There is a sequence $(D_k)_{k=0,\dots,m}$ such that*
   (a) $\biguplus_{k=0,\dots,m} D_k = D$
   (b) $\forall k = 0, \dots, m\ E_w(l, D_k, H, k)$

Proof: First, we assume that $E_w(l, D, H)$. If $\exists x(x \to_r u^i \,\&\, x \to_r v^j)$, then Lemma 6.7 implies $u^i \sim_0 v^j$ or $\exists k \in \mathrm{DP}\ u^i \sim_k v^j$. Let $k \in \mathrm{DP}$. Then we set $\mathcal{U}_{D_k} := \{u \in \mathcal{U}_D \mid \forall i, j \in H\, l_i(u) \sim_k l_j(u)\}$. By Lemma 8.4 it follows that the $\mathcal{U}_{D_k}$s are pairwise disjoint. It shows also that $\langle \mathcal{U}_{D_K}, \mathrm{Con}_{D_k}\rangle$ with $\mathrm{Con}_{D_k} := \{\varphi \in \mathrm{Con}_D \mid FV(\varphi) \subseteq \mathcal{U}_{D_k}\}$ is a proper DRS, and that $\biguplus_{k\in\mathrm{DP}} D_k = D$.

Let $(D'_k)_{k=1,\dots,m}$ be any other decomposition of D. Then it follows by $E_w(l, D'_k, k)$ that $\mathcal{U}_{D'_k} \subseteq \{u \in \mathcal{U}_D \mid \forall i, j \in H\, l_i(u) \sim_k l_j(u)\} = \mathcal{U}_{D_k}$. By $\biguplus_{k\in\mathrm{DP}} D'_k = D$ it follows that $D_k = D'_k$ for all $k \in \mathrm{DP}$.

We define $\mathcal{U}_{D_0} := \{u \in \mathcal{U}_D \mid \forall i \in H\, l_i(u) \in \mathrm{Obs}_{D_i}\}$. For $|H| = 1$, $H = \{i\}$ it follows that $\mathcal{U}_{D_0} = \mathrm{dom}\,\mathrm{Obs}_{D_i}$. If $|H| > 1$, then Lemma 6.7 shows that $\mathcal{U}_{D_0}$ is disjoint from all $\mathcal{U}_{D_k}$, and that $\bigcup_{k=0,\dots,m} \mathcal{U}_{D_k} = \mathcal{U}_D$.

The third condition follows trivially from the second. Hence assume that the third property holds. We need to define suitable functions $f : \mathcal{U}_{l_{i/j}(D)} \longrightarrow \mathcal{U}_{D_{j/i}}$. But we can get them easily from the definitions of $E_w(l, D_k, H, k)$. $\square$

In the next two lemmas we take a closer look at observations. Let D be a DRS which is common for a group H relative to some l, i.e. $C_w(l, D, H)$, and where the referents are jointly chained to the same objects in the described situation, i.e. $E_w(l, D, H, 0)$. Then, the two lemmas together show that there is exactly one observation–act by which all the $l_i(D)$ have been introduced into the local states of the participants $i \in H$.

Lemma 8.7 *Let s be an initial state, $a \in \mathcal{A}(s)$ and $w = \gamma_s(a)$. Let (D, l) be such that $\forall n \in \mathbf{N}\, E^n_w(l, D, H, 0)$ for a group $H \subseteq \mathrm{DP}$. Let $\varphi \in \mathrm{Con}_D$, $u \in FV(\varphi)$. Then there is an act* `act` *in a, $H' \supseteq H$, D', l' and a $\varphi' \in \mathrm{Con}_{D'}$ such that for all $j \in H$ $\varphi'/_s l' = \varphi/_s(\mathrm{Obs}_{D_j} \circ l_j)$ and* $\mathtt{act}_j$ *has the form* $\mathtt{observe}(H', D', l')$.

Proof: The claim clearly holds for $|H| = 1$.

Case 1: $H = \{i, j\}$. Assume that there is no act $\mathtt{act}$ in a with the desired properties. There is a joined act in a with $\mathtt{act}_i$ of the form $\mathtt{observe}(H', D', l')$ such that there is $\varphi' \in \mathrm{Con}_{D'}$ where for i $\varphi'/_s l' = \varphi/_s(\mathrm{Obs}_{D_i} \circ l_i)$. Because otherwise $\varphi/_s l_i$ can't be an element of Con_{D_i}.

To simplify our proof we concentrate on the case where $|FV(\varphi)| = 1$. By our assumption it follows that for all acts $\mathtt{act}$ in a of the form $\mathtt{observe}(H', D', l')$ we have $i \in H' \Rightarrow j \notin H'$. Let $i \in H'$. (1) Assume further that there is no $\mathtt{act}'$ in a such that $\mathtt{act}'_i$ is of the form $\mathtt{send}(i, H'', D'', l'')$ where φ is represented in $\mathrm{Con}_{D''}$ and $j \in H''$. Then let a' be the sequence of actions where all acts $\mathtt{act}$ of a where $\mathtt{act}_i$ is of the form $\mathtt{observe}(H', D', l')$ and where φ is represented in D' are removed. Then, $\gamma_s(a')$ is indiscernible from w for j. But φ can't be represented in $D_i^{\gamma_s(a')}$ in such a way that the free variables are elements of $\mathrm{Obs}_{D_i^{\gamma_s(a')}}$, therefore D can't be common. (2) Hence, assume that there are $\mathtt{act}^{1/2}$ in a such that $\mathtt{act}^1_i$ is of the form $\mathtt{send}(i, H^1, D^1, l^1)$ where φ is represented in Con_{D^1} and $j \in H^1$. $\mathtt{act}^2_j$ should be of the related form for j instead of i and 2 instead of 1. Assume that $\mathtt{act}^2$ is later than $\mathtt{act}^1$. Let v be the variable introduced by $\mathtt{act}^1$ into $\mathcal{U}_{D_j}$ for φ. Then define $\tilde{l}$ in the same way as l^2 except for the variables v' in the representation of φ. Here we define $\tilde{l}(v') := v$. Then $\mathtt{send}(j, H^2, D^2, \tilde{l})$ would have been a possible action for j in the situation where he performed $\mathtt{act}^2$. Hence we may replace $\mathtt{act}^2$ by this action in a and get a sequence a'. But then $\gamma_s(a')$ is for i indiscernible from w. We can additionally eliminate all acts with $\mathtt{observe}(H', D', l')$ where D' represents φ and $j \in H'$. For this new sequence a'' we find also that $\gamma_s(a'')$ is for i indiscernible from w. But φ can't be represented in $D_j^{\gamma_s(a'')}$ in a way such that the free variables are elements of $\mathrm{Obs}_{D_j^{\gamma_s(a'')}}$. This contradicts the assumption.

Case 2: $|H| = n + 1$. We choose some $i \in H$. Let H' be any set of participants such that $i \notin H'$ and where there is an act $\mathtt{act}$ with $\mathtt{act}_j$ of the form $\mathtt{observe}(H', D', l')$, $j \in H'$. But then, we can replace these acts by sequences of acts $\mathtt{act}$ with $\mathtt{act}_j$ of the form $\mathtt{observe}(\{j'\}, D', l')$, $j' \in H'$. Thereby we get a new sequence a'. We find that for i $\gamma_s(a)$ and $\gamma_s(a')$ are indiscernible. But by induction it follows that D can't be common for H' in $\gamma_s(a')$.

All other cases follow, if we interchange i and j. $\square$

Lemma 8.8 *Let (D, l) be such that $C(l, D, H)$ and $E_w(l, D, H, 0)$ for a group $H \subseteq \mathrm{DP} = \{1, \ldots, m\}$ and a possibility w. Then $\forall n \in \mathbf{N}\, E^n_w(l, D, H, 0)$.*

Proof: If $|H| \leq 1$, then the claim is trivial. Hence, let $|H| > 1$. Let $i \in H$. Let v be an indiscernible possibility for i in w. Then, there is an l^v with $E(l^v, D, H)$, $l^v_i = l^w_i$ and $\forall u \in \mathcal{U}_D\, \forall j, j' \in H\, \forall k \geq 0 (l^w_j(u)^j \sim_k l^w_{j'}(u)^{j'} \Leftrightarrow l^v_j(u)^j \sim_k l^v_{j'}(u)^{j'})$. By Lemma 8.6 there is a sequence $(D_k)_{k=0,\ldots,m}$, such that $\biguplus_k D_k = D$ and $\forall k = 0, \ldots, m\, E_v(l^v, D_k, H, k)$. Suppose that $\mathcal{U}_{D_k} \neq \emptyset$ for a $k \neq 0$. Then it holds for all $j \in H \setminus \{i\}$ and $u \in \mathcal{U}_{D_k}$ that $l^v_i(u)^i \sim_k l^v_j(u)^j$ and $l^w_i(u)^i \not\sim_0 l^w_j(u)^j$, which contradicts the properties of l^v stated above. Therefore, $E(l^v, D, H, 0)$. By induction it follows then that for all $n \in \mathbf{N}$ $E^n(l^v, D, H, 0)$.
$\square$

In the next two lemmas we show the existence of two useful partitions of common DRSes.

Lemma 8.9 *Let s be an initial state, $a \in \mathcal{A}(s)$ and $w = \gamma_s(a) \in \mathcal{W}(\mathcal{G})$. Let (D, l) and H be such that $C_w(l, D, H)$.*

If there is an $k \in \mathrm{DP}$ with $E_w(l, D, H, k)$, then there is a sequence $(D_r)_{r \leq lh(a)}$ such that:

1. *$\biguplus_{r \leq lh(a)} D_r = D$,*
2. *if D_r is not empty, then it holds for all $i \in H$ that for the r–th act $\mathtt{act}$ in a $\mathtt{act}_i$ is of the form $\mathtt{get}(k, H', D')$ with $H' \supseteq H$ and $D' \trianglelefteq_f D_r$ for an injective f.*

It follows that $E_w(l, D, H, k)$ implies $\forall n \in \mathbf{N}\, E^n_w(l, D, H, k)$, and that $l_i : \mathcal{U}_D \longrightarrow \mathcal{U}_{D_i}$ is injective for all $i \in H \setminus \{k\}$.

Proof: If $H \subseteq \{i, k\}$, then the claim follows from Lemma 6.7. Let H contain at least two agents different from k. By Lemma 6.9 it follows that for all $\varphi \in \mathrm{Con}_D$, $i \in H \setminus \{k\}$ there are D^i, H^i, and $\mathtt{act}^i$ such that $(\mathtt{act}^i)_i$ is of the form $\mathtt{get}(k, D^i, H^i)$, D^i represents φ, $i \in H^i$, and such that $\mathtt{act}^i$ introduced $l_i(u)$ for $u \in FV(\varphi)$ into D_i. We denote by $l_k(u)$ the unique subject in D_k such that for all $i \in H \setminus \{k\}$ $l_k(u) \to l_i(u)$. We show that $\mathtt{act}^i = \mathtt{act}^j$ for $i, j \in H \setminus \{k\}$. We first show that $H^i \supseteq H \setminus \{k\}$. Suppose that there are $i, j \neq k$ such that for all such acts necessarily $i \notin H^j$ and $j \notin H^i$. Let $u_0 \in FV(\varphi)$.

If there is a later $\mathtt{send}$–act by i which depend on the material introduced by $\mathtt{act}^i$ into his local DRS, then we can include an appropriate $\mathtt{observe}$–act of the form $\mathtt{observe}(\{i\}, D', l')$ which too could have introduced the relevant material. Lets name this sequence of actions a'. For j we find that $\gamma_s(a)$ and $\gamma_s(a')$ are indiscernible. We then may remove $\mathtt{act}^i$ from a', and get a sequence a'' such that for j $\gamma_s(a'')$ is indiscernible from $\gamma_s(a')$ and $\gamma_s(a)$. But for $\gamma(a'')$ there is no referent u_1 in the local DRS of i such that there is a referent u_2 in k's local state such that $u^k_2 \to u^i_1$ and $u^k_2 \to l_j(u)^j$. Hence, the condition of Def 8.2 for $C_w(l, D, H)$ is violated. Therefore, we find $H^i \supseteq H \setminus \{k\}$.

Now assume that $l_{i/j}(u)$ was introduced by $\mathtt{act}^{i/j}$ into $D_{i,j}$, and that $\mathtt{act}^i \neq \mathtt{act}^j$. Assume that $\mathtt{act}^j$ was later. Then, we can add to the sequence a an act $\mathtt{act}$ where i repeats for k what he has heard from k. Hence, this act introduces new subjects (only) into the local DRS D_k. Then, we can replace $\mathtt{act}^j$ by an act $\mathtt{act}'$ where $\mathtt{act}'_k$ is of the form $\mathtt{send}(k, D^j, H^j, l')$, and where l' chains the referents in $\mathcal{U}_D$ to the referents introduced by $\mathtt{act}$ into $\mathcal{U}_{D_k}$. We call the new sequence a'. $\gamma_s(a)$ and $\gamma_s(a')$ are indiscernible for j. But for $\gamma(a')$ there is no referent u_1 in the local DRS of i such that there is a referent u_2 in k's local state such that $u^k_2 \to u^i_1$ and $u^k_2 \to l_j(u)^j$. Hence, the condition of Def 8.2 for $C_w(l, D, H)$ is violated again.

Lemma 6.9 shows that $\mathtt{act}^i$ is unique, and that $\varphi/_s l_i$ and all free variables in $FV(\varphi/_s l_i)$ have been introduced by this same act. This allows us to define D_r as the union of all D_φ such that $\varphi/_s l_i$ was introduced into the local state of i by the r–th act for some $i \in H \setminus \{k\}$.

Finally, we have to show that there exists an injective f such that $D' \trianglelefteq_f D_\varphi$ where $\varphi/_s l_i$ was introduced into the local DRS D_i by an act with $\mathtt{act}_i = \mathtt{get}(k, H', D')$, $H' \supseteq H$. Let $u_1 \neq u_2 \in FV(\varphi)$. Hence, there exists $k' \in H$ such that $l_{k'}(u_1) \neq l_{k'}(u_2)$. If $k' \neq k$, then f exists according to Lemma 6.10. Hence, let $k' = k$. But $\mathtt{send}(k, H', D', l')$ is licensed only if l' is injective. Hence, we can find an injective f with $D' \trianglelefteq_f D_\varphi$. By definition of the transition operation τ it follows that for all $H \ni i \neq k$ $l_i : \mathcal{U}_D \longrightarrow \mathcal{U}_{D_i}$ is injective. $\Box$

Lemma 8.10 *Let D be a DRS, $w = \langle \mathfrak{A}, \rightarrow, \langle D_1, I_1 \rangle, \dots, \langle D_m, I_m \rangle \rangle \in \mathcal{W}(\mathcal{G})$. Let $H \subseteq \mathrm{DP}$, $\mathrm{DP} = \{1, \dots, m\}$. Let (D, l) be such that $C_w(l, D, H)$.*

Then there exists a partition $(D_k)_{k=0,\dots,m}$ of D such that $\forall k = 0, \dots, m \, \forall n \in \mathbf{N} \, E^n_w(l, D_k, H, k)$.

Proof: By Lemma 8.6 there is partition $(D_k)_{k=0,\dots,m}$ of D such that $\forall k = 0, \dots, m \, E_w(l, D_k, H, k)$. The claim follows then from Lemma 8.8 and Lemma 8.9. □

Now we can proof the central result.

Theorem 8.11 *Let s be an initial state, $a \in \mathcal{A}(s)$ and $w^{(+)} := \gamma^{(+)}_s(a)$ in $\mathcal{W}(\mathcal{G}^{(+)})$. We set*

$$l^w := \left(l^{w^+}_{i,H} \right)_{i \in \mathrm{DP}}.$$

Then, we find:

1. $C_w\left(l^w, D^{w^+}_H, H\right)$,
2. *If $C_w(l, D, H)$ for any l, D, then the function $f : \mathcal{U}_D \longrightarrow \mathcal{U}_{D^{w^+}_H}$ defined by*

$$f(u) = u' \text{ iff } \exists i \in H \, l_i(u) = l^w_i(u')$$

is an injection, and $f(D) \subseteq D^{w^+}_H$.

Proof: As $D^w_i = D^{w^+}_{i,\{i\}}$ it follows that l^w is well–defined. If $l^w_i(u) = u' \in \mathcal{U}_{D_i}$, then u' was introduced into D_i by an `act` of the form `get`(k, H', D'), $H' \supseteq H \setminus \{k\}$, or `observe`$(H', D', l')$, $H' \supseteq H$. Furthermore, if $u \in FV(\varphi)$, and $\varphi \in Con_{D^{w^+}_H}$, then $\varphi/_s l^w_i$ was introduced by the same act together with all other free variables. Hence, the local part of `act` is part of the local states for all $i \in H$. It follows from the definition of the games in $\mathcal{G}$ that the same act is part of all local states in $CG_H(w)$ (of course, the anchoring function l' in `send`(k, H', D', l') for the speaker k may vary). This allows us to define simultaneously the embedding functions l^v for all $v \in CG_H(w)$. We can prove then that for all v $E^v(l^v, D, H)$, and that the conditions of Definition 8.2 hold. This proves the first part of the theorem.

Let (D, l) be such that $C_w(l, D, H)$. Lemma 8.10 shows that there is a decomposition $(D_k)_{k \geq 0}$ of D such that for all $k \geq 0$ and $n \in \mathbf{N}$ $E^n_w(l, D_k, H, k)$, and such that $\biguplus_{k \geq 0} D_k = D$. For $k = 0$ Lemma 8.7 shows that for every $\varphi \in Con_D$ there is an act in a where $\mathtt{act}_i$ is of the form `observe`(H', D', l') for $i \in H \subseteq H'$ and such that $\varphi'/_s l' = \varphi/_s(\mathrm{Obs}_{D_i} \circ l_i)$. Hence, there must be an act where $\mathtt{act}_i$ is of the form `observe`(H'', D'', l'') for $i \in H \subseteq H''$ and where $\varphi'/_s l' = \varphi''/_s l''$, and which introduced φ''' into $D^{w^+}_H$ with $\varphi'''/_s(\mathrm{Obs}_{D_i} \circ l^{w^+}_{i,H}) = \varphi''/_s l''$. This shows that $f(u) := u'$ *iff* $\exists i \in H \, l_i(u) = l^w_i(u')$ is an injection on $\mathcal{U}_{D_0}$ with $f(D_0) \subseteq D^{w^+}_H$.

Let $(D_r)_{r \leq lh(a)}$ be the sequence constructed in Lemma 8.9. If D_r is not empty, then the r–th act `act` of a has the property that for some $i \in H$ $\mathtt{act}_i$ is of the form `get`(k, H', D'), $H' \supseteq H \setminus \{k\}$ with $D' \trianglelefteq_f D_r$ for an injective f. We find for $u \in \mathcal{U}_D$ a unique u_0 in some D' such that for all $i \in H$ $l_i(u)$ is the discourse referent introduced by u_0. We find also that $u \in FV(\varphi)$, $\varphi \in \mathrm{Con}_D$, implies that $\varphi/_s l_i$ was introduced into the local DRS of i by the same act. But this act introduces for u_0 and φ exactly one referent u' into D_H^{w+}, and l_i^w maps u' back to the referent introduced by u_0 into D_i^w. Hence, $l_i(u) = l_i^w(u')$, for each $u \in \mathcal{U}_D$ there is exactly one such u', and it is the same for all $i \in H$. As distinct referents in D' introduce distinct referents into $D_H^{w^+}$, it follows that $f(u) := u'$ *iff* $l_i(u) = l_i^w(u')$ defines an injective function. Lemma 6.10 and the construction of τ for $\mathcal{G}^+$ shows also that $f(D) \subseteq D_H^{w^+}$. □

9 Final Remarks and Conclusions

In Section 4 we promised to show the following properties of common DRSes and their internal representations:

1. The set of common DRSes has maximal elements.
2. The maximal common DRSes are identical up to substitutions of variables.
3. The representing DRS for a group H is a maximal common DRS for this group.

We can now make these statements precise. Let s be an initial situation, $a \in \mathcal{A}(s)$ a sequence of actions, $H \subseteq$ DP a group, and $w = \gamma_s(a)$, $w^+ = \gamma_s^+(a)$. Then, let $\mathcal{C}_w(H) := \{D \in \text{DRS} \mid C_w(D, H)\}$ be the set of all common DRS for group H in w, and $D \leq D'$ *iff* $\exists f : \mathcal{U}_D \longrightarrow \mathcal{U}'_D$ inj. : $f(D) \subseteq D'$. $\leq$ is a pre–order on $\mathcal{C}_w(H)$. It defines a order on the equivalence classes $[D] := \{D' \in \mathcal{C}_w(H) \mid D \leq D' \;\&\; D' \leq D\}$. Then:

1. $(\mathcal{C}_w(H), \leq)$ has maximal elements.
2. The maximal elements are unique up to substitution of variables, i.e. for maximal DRSes D and D' there are bijective functions $f : \mathcal{U}_D \longrightarrow \mathcal{U}_{D'}$ such that $f(D) = D'$.
3. $D_H^{w^+}$ is a maximal DRS in $(\mathcal{C}_w(H), \leq)$.

This follows directly from Theorem 8.11. Furthermore, we should note that:

1. $\forall v \in CG_w(H)\, CG_v(H) = CG_w(H)$.
2. $\forall v \in \mathcal{W}(\mathcal{G}) : v \in CG_w(H) \Rightarrow \mathcal{C}_v(H) = \mathcal{C}_w(H)$.
3. $\forall v^+ \in \mathcal{W}(\mathcal{G}^+) : v^+ \in CG_{w^+}(H) \Rightarrow D_H^{v^+} = D_H^{w^+}$.
4. $\forall s', a' : \gamma_{s'}(a') \in CG_w(H) \Rightarrow \gamma_{s'}^+(a') \in CG_{w^+}(H)$.

The directions $\Leftarrow$ don't hold in general. This is due to the additional information about the actions in a which is available in the local states in the games in $\mathcal{G}$.

The proofs in the last sections depend very much on the concrete definitions of the multi–agent systems for $\mathcal{G}$ and $\mathcal{G}^+$. Hence, it is dubious whether the results generalise for more complex and restricted fragments of dialogue. Our fragment allows e.g. for repeated statements of the same facts. This allows for the introduction of a lot of superfluous discourse referents. E.g. assume that the described situation contains only one object, and that everybody who observes this scene knows that it must be common knowledge for all observers that there is only this one. Assume it is a broken drinking glass. Then, repeated statements of *there is a broken drinking glass* will introduce for every utterance a new referent which represents this glass into the common DRSes. But the available visual information will make clear that there is only one broken drinking glass:

(6) Anne enters a room. Last night, there has been a party. There is a table, and on the table there is a broken drinking glass.

Anne: *I have just been in the room there. Have you been there too? Did you have a look at the table?*

Bob: *Yes, I did.*

Anne: The broken drinking glass *is there from the party last night.*

Anne's use of *the broken drinking glass* seems to be ok. But, of course, no common discourse referent is available at this point of time. This shows a clear limitation of our approach. There are, of course, more sources of additional information which can not be handled by our theory.

Conclusions

We are able to represent the chains that are defined by iterated specific uses of indefinite NPs. The theory of multi–agent systems, which builds the basis for our model, provides us with natural descriptions of the common ground as an information state representing mutual information. In order to explain the referential anaphoric use of a definite description, and especially how to apply it's uniqueness condition, we found that this use is sensitive to common substructures of the local states of discourse participants. We characterised them as common DRSes and explained how the participants can represent these common DRSes.

- Specifically used indefinite NPs introduce *free* variables. The interpretation function, which is necessary to define the truth values for the conditions of a DRS, are provided by an external chain relation. They never get existentially bound.
- There are three distinct objects in our model which are possible representations for the linguistic *common ground*: (1) The information states representing common knowledge, (2) the common DRSes, and (3) the internal representations of the common DRSes.
- The uniqueness condition connected to an anaphorically used definite description does not contribute to the *asserted meaning* of a sentence. If a speaker uses a description of the form $\text{def}x.\varphi(x)$, then this use *presupposes* that there is an object a such that $\varphi(a)$ holds.

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